AP Calculus BC

Topic: Antidifferentiation by Parts

Instructions

Solve the following problems. Show all your work clearly and use the formula for integration by parts:

Practice Problems

1. **Basic Integration by Parts:** Evaluate the following integrals using integration by parts:

    (i) $\int xe^x dx$

    (ii) $\int x \ln(x) dx$

    (iii) $\int x \cos(x) dx$

    (iv) $\int e^x \sin(x) dx$

    (v) $\int \ln(x) dx$

2. **Advanced Integration by Parts:** Solve the following integrals:

    (i) $\int x^2 e^x dx$

    (ii) $\int x^2 \ln(x) dx$

    (iii) $\int e^x \cos(x) dx$

    (iv) $\int x^2 \sin(x) dx$

3. **Repeated Integration by Parts:** Evaluate the following using repeated application of integration by parts:

    (i) $\int x^3 e^x dx$

(ii) $\int x^2 \ln(x)dx$

4. **Definite Integrals:** Evaluate the following definite integrals using integration by parts:

   (i) $\int_0^1 xe^x dx$

   (ii) $\int_1^e \ln(x)dx$

   (iii) $\int_0^{\pi/2} x\cos(x)dx$

5. **Applications:** The work done by a variable force $F(x) = xe^x$ (in N) acting over a distance $[0, 2]$ (in m) is given by:

$$W = \int_0^2 F(x)dx.$$

Find the work done.

Multiple Choice Questions

1. Evaluate $\int xe^x dx$ using integration by parts.

   a. $e^x(x-1) + C$

   b. $e^x(x+1) + C$

   c. $xe^x - e^x + C$

   d. $xe^x + e^x + C$

2. Which of the following is the integral of $\int x^2 \ln(x)dx$?

   a. $\frac{x^3}{3}\ln(x) - \frac{x^3}{9} + C$

   b. $\frac{x^3}{3}\ln(x) + \frac{x^3}{9} + C$

   c. $\frac{x^3}{3} + \ln(x) + C$

   d. $x^2 \ln(x) - x^2 + C$

3. Evaluate $\int_0^1 x\ln(x)dx$ using integration by parts.

   a. $-\frac{1}{4}$

   b. $\frac{1}{2}$

   c. $-\frac{1}{2}$

   d. 1