AP Calculus BC

Topic: Antidifferentiation by Substitution

Instructions

Solve the following problems. Show all your work clearly and include all substitution steps.

Practice Problems

1. **Basic Substitution:** Evaluate the following integrals using u-substitution:
   (i) $\int (2x+3)^4 dx$
   (ii) $\int x e^{x^2} dx$
   (iii) $\int \frac{x}{\sqrt{1+x^2}} dx$
   (iv) $\int \sin(2x) dx$
   (v) $\int \cos^3(x)\sin(x) dx$
2. **Definite Integrals:** Evaluate the following definite integrals using substitution:
   (i) $\int_1^4 2x\sqrt{x^2+1} dx$
   (ii) $\int_0^{\pi/2} \cos(2x) dx$
   (iii) $\int_0^1 \frac{e^{x^2}}{2x} dx$
   (iv) $\int_1^2 \frac{\ln(x)}{x} dx$
   (v) $\int_0^1 (x^3+1)^5(3x^2) dx$
3. **Advanced Substitution:** Solve the following integrals:
   (i) $\int (3x^2+1)e^{x^3+x} dx$

(ii) $\int \frac{\sin(x)}{1+\cos^2(x)}dx$

(iii) $\int (x^2+4)^3(2x)dx$

(iv) $\int \frac{x^2}{(1+x^2)^2}dx$

4. **Applications:**

   (i) The velocity of a particle is given by $v(t) = te^{t^2}$. Find the total displacement of the particle over the interval $[0, 1]$.

   (ii) The rate of water flow into a tank is $R(t) = \frac{5}{1+t^2}$. Find the total amount of water that flows into the tank from $t = 0$ to $t = 2$.

Multiple Choice Questions

1. Evaluate $\int x(x^2+1)^3dx$.

   a. $\frac{1}{4}(x^2+1)^4 + C$

   b. $\frac{1}{3}(x^2+1)^3 + C$

   c. $\frac{1}{2}(x^2+1)^2 + C$

   d. $(x^2+1)^3 + C$

2. Find the value of $\int_0^1 (3x^2+1)(x^3+x)dx$ using substitution.

   a. 2

   b. 3

   c. 4

   d. 5

3. Which substitution would you use to evaluate $\int xe^{x^2}dx$?

   a. $u = x^2$

   b. $u = e^{x^2}$

   c. $u = xe^x$

   d. $u = \ln(x)$

4. Evaluate $\int_0^{\pi/2} \sin(2x)dx$.

   a. 0.5

   b. 1

   c. 2

   d. 0