AP Calculus BC

Topic: Exponential Growth and Decay

Instructions

Solve the following problems. Show all your work clearly and include units in your answers where appropriate.

Practice Problems

1. **Basic Exponential Growth and Decay:** Solve the following differential equations:
   (i) $\frac{dy}{dt} = 3y$, with $y(0) = 5$.
   (ii) $\frac{dy}{dt} = -2y$, with $y(0) = 10$.
   (iii) $\frac{dP}{dt} = 0.1P$, with $P(0) = 100$.
   (iv) $\frac{dN}{dt} = -0.3N$, with $N(0) = 50$.
2. **Real-World Applications:**
   (i) The population of a city grows at a rate proportional to its size. If the population doubles in 5 years, find the exponential growth model for the population $P(t)$.
   (ii) A radioactive substance decays at a rate proportional to its mass. If 20
   (iii) The amount of bacteria in a culture grows exponentially. If the initial population is 200 and the population triples in 4 hours, find the population after 10 hours.
   (iv) A certain chemical decays at a rate of 15
3. **Definite Integrals in Growth and Decay:** Solve the following:

(i) The rate of change of a population is given by $\frac{dP}{dt} = 5e^{0.2t}$. Find the total change in the population over the interval $[0, 10]$.

(ii) The rate of decay of a substance is given by $\frac{dM}{dt} = -3e^{-0.5t}$. Find the total amount decayed over the interval $[0, 5]$.

4. **Half-Life Problems:**

(i) The half-life of a radioactive substance is 8 hours. If the initial mass is 50 grams, find the amount remaining after 20 hours.

(ii) A certain isotope has a half-life of 5 years. If the initial mass is 10 grams, how long will it take for the mass to decay to 1 gram?

Multiple Choice Questions

1. Solve $\frac{dy}{dt} = 4y$, with $y(0) = 2$.

   a. $y = 2e^{4t}$

   b. $y = 4e^{2t}$

   c. $y = 2e^{2t}$

   d. $y = 4e^{4t}$

2. The population of a town grows at a rate proportional to its size. If the initial population is 500 and it doubles in 6 years, what is the population after 12 years?

   a. 1000

   b. 2000

   c. 1500

   d. 2500

3. A radioactive substance decays at a rate proportional to its mass. If 60

   a. 36%

   b. 30%

   c. 25%

   d. 40%

4. A bacteria culture grows exponentially. If the initial population is 100 and it triples in 4 hours, what is the population after 8 hours?

   a. 300

   b. 900

   c. 600

   d. 1200