



AP Calculus BC

Topic: Improper Integrals

Instructions

Solve the following problems. Show all your work clearly.

Practice Problems

1. Evaluate the following improper integrals:

- (i) $\int_1^{\infty} \frac{1}{x^2} dx$
- (ii) $\int_0^1 \frac{1}{\sqrt{x}} dx$
- (iii) $\int_1^{\infty} e^{-x} dx$

2. Determine if the following integrals converge or diverge:

- (i) $\int_0^{\infty} \frac{1}{x+1} dx$
- (ii) $\int_1^{\infty} \ln(x) dx$
- (iii) $\int_0^{\infty} e^{-x^2} dx$

3. Break down and evaluate the following improper integrals:

- (i) $\int_{-\infty}^{\infty} \frac{1}{x^2+1} dx$
- (ii) $\int_1^{\infty} \frac{x}{x^2+1} dx$
- (iii) $\int_0^{\infty} \frac{1}{x^p} dx$ for $p > 1$

4. Compare the convergence of the following integrals:

- (i) $\int_1^{\infty} \frac{1}{x^2} dx$ and $\int_1^{\infty} \frac{1}{x} dx$
- (ii) $\int_1^{\infty} \frac{1}{x^p} dx$ for $p = 2$ and $p = 1.5$
- (iii) $\int_0^{\infty} e^{-x} dx$ and $\int_0^{\infty} e^{-x^2} dx$

Challenge Problem

1. Prove that $\int_0^\infty \frac{\sin(x)}{x} dx$ converges, but does not have a simple closed-form solution.

Multiple Choice Questions

1. Does the integral $\int_1^\infty \frac{1}{x^p} dx$ converge when $p = 1$?
 - a. Yes
 - b. No
 - c. Converges for $p < 1$
 - d. None of the above
2. What is $\int_1^\infty \frac{1}{x^2} dx$?
 - a. ∞
 - b. 1
 - c. $\frac{1}{2}$
 - d. 0
3. Which of the following integrals diverges?
 - a. $\int_1^\infty \frac{1}{x^2} dx$
 - b. $\int_0^1 \frac{1}{\sqrt{x}} dx$
 - c. $\int_1^\infty \ln(x) dx$
 - d. $\int_0^\infty e^{-x} dx$
4. Evaluate $\int_0^\infty e^{-x} dx$:
 - a. 1
 - b. 0
 - c. ∞
 - d. Does not exist

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