

AP Calculus BC

Topic: L'Hôpital's Rule

Instructions

Solve the following problems. Show all your work clearly.

Practice Problems

- Evaluate the following limits using L'Hôpital's Rule:

(i) $\lim_{x \rightarrow \infty} \frac{x}{e^x}$

(ii) $\lim_{x \rightarrow 0^+} \frac{\ln(x)}{1/x}$

(iii) $\lim_{x \rightarrow 0} \frac{\sin(x)}{x}$

- Determine whether L'Hôpital's Rule applies and evaluate the limit if possible:

(i) $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$

(ii) $\lim_{x \rightarrow \infty} \frac{\sqrt{x}}{x}$

(iii) $\lim_{x \rightarrow 0} \frac{e^x - 1}{x}$

- Find the horizontal asymptotes of the following functions using limits:

(i) $f(x) = \frac{2x}{x^2 + 1}$

(ii) $f(x) = \frac{x^2}{e^x}$

(iii) $f(x) = \frac{\ln(x)}{x}$

- Evaluate the following indeterminate forms using L'Hôpital's Rule:

$$(i) \lim_{x \rightarrow \infty} \frac{x^3}{e^x}$$

$$(ii) \lim_{x \rightarrow 0^+} x \ln(x)$$

$$(iii) \lim_{x \rightarrow \infty} \frac{\ln(x)}{\sqrt{x}}$$

Challenge Problem

1. Prove that $\lim_{x \rightarrow 0} \frac{e^x - 1 - x}{x^2} = \frac{1}{2}$ using L'Hôpital's Rule.

Multiple Choice Questions

1. What is $\lim_{x \rightarrow 0} \frac{\sin(2x)}{x}$?

- a. 0
- b. 2
- c. 1
- d. Does not exist

2. Evaluate $\lim_{x \rightarrow \infty} \frac{\ln(x)}{x}$:

- a. 0
- b. 1
- c. ∞
- d. Does not exist

3. If $f(x) = \frac{e^x}{x}$, find $\lim_{x \rightarrow \infty} f(x)$:

- a. 0
- b. 1
- c. ∞
- d. Does not exist

4. What is $\lim_{x \rightarrow 0} \frac{\ln(1+x)}{x}$?

- a. 1
- b. 0
- c. ∞
- d. Does not exist