AP Calculus BC

Topic: Lengths of Curves

Instructions

Solve the following problems. Show all your work clearly and include units in your answers where appropriate.

Practice Problems

1. **Find the length of the curve given by the following functions:**
   i. $y = x^2$ for $x \in [0, 2]$.
   ii. $y = \sqrt{x}$ for $x \in [1, 4]$.
   iii. $y = \ln(x)$ for $x \in [1, e]$.
2. **Parametric Curves: Find the arc length of the following parametric curves:**
   i. $x = t^2,\ y = t^3$, for $t \in [0, 1]$.
   ii. $x = \cos(t),\ y = \sin(t)$, for $t \in [0, \pi/2]$.
3. **Applications of Arc Length:**
   i. Find the length of the curve $y = \sin(x)$ for $x \in [0, \pi]$.
   ii. Find the length of the curve $y = e^x$ for $x \in [0, 1]$.

Multiple-Choice Questions

(a) The arc length of $y = x^2$ on $[0, 1]$ is:

a. $\sqrt{5}$
b. $\frac{\sqrt{5}}{2}$
c. $\frac{3\sqrt{5}}{4}$
d. $\frac{\sqrt{5}}{3}$

(b) The arc length of $y = \ln(x)$ on $[1, e]$ is:

a. $\ln(e)$
b. $\sqrt{2}$
c. $\sqrt{5}$
d. $\ln(e) + 1$

(c) The arc length of the parametric curve $x = \cos(t),\ y = \sin(t)$ on $[0, \pi/2]$ is:

a. $\frac{\pi}{2}$
b. 1
c. π
d. $\frac{\pi}{4}$

Challenge Problems

1. Find the length of the curve $y = \sqrt{x^3 + 1}$ for $x \in [0, 2]$.