AP Calculus AB

Topic: Modeling and Optimization

Instructions

Solve the following problems. Show all work and include proper mathematical justifications.

Practice Problems

1. A rectangle has a perimeter of 20 meters. Find the dimensions of the rectangle that maximize its area.

2. A cylindrical can is to be constructed to hold 1 liter of liquid. Find the dimensions of the can (radius and height) that minimize the surface area.

3. A farmer wants to fence off a rectangular field along a straight river. The farmer has 300 meters of fencing and needs no fence along the river. Find the dimensions of the field that maximize its area.

4. A company manufactures open-top boxes by cutting equal squares from the corners of a 12-inch by 12-inch piece of cardboard and folding up the sides. What size squares should be cut out to maximize the volume of the box?

5. Find the dimensions of a rectangular box with a square base and no top that has a volume of 32 cubic units and minimizes the surface area.

6. A car travels 100 miles in 2 hours. The speed of the car varies during the trip. Using calculus, determine the minimum possible average speed for at least one moment in time according to the Mean Value Theorem.

Multiple Choice Questions

1. What is the derivative of the area function $A(x) = x(20 - 2x)$?

   a. $20 - 4x$

   b. $10 - x$

   c. $20x - 4x^2$

   d. None of the above

2. For the open-top box problem, the volume function is $V(x) = x(12 - 2x)^2$. What is the critical point of $V(x)$?

   a. $x = 2$

   b. $x = 3$

   c. $x = 4$

   d. None of the above

3. A company wants to minimize the cost of constructing a box with a square base and no top. If the volume is fixed, the cost is minimized when:

   a. The base area equals the height.

   b. The base area is twice the height.

   c. The base area is half the height.

   d. The base area equals the surface area.

4. For a cylinder with fixed volume V, the surface area is minimized when:

   a. The height equals the diameter.

   b. The radius equals the height.

   c. The height is twice the radius.

   d. None of the above