AP Calculus BC

Topic: Parametric Functions

Instructions

Solve the following problems involving parametric equations. Eliminate parameters where required, compute derivatives, and sketch graphs where applicable. Show all work for full credit.

Practice Problems

1. **Sketch the parametric curves and eliminate the parameter to express y as a function of x:**

   (i) $x = 3t + 1$, $y = 2t - 4$ for $t \in [0, 4]$.

   (ii) $x = \sqrt{t-1}$, $y = \frac{t-2}{3}$ for $t \in [1, 5]$.

   (iii) $x = \tan(t)$, $y = \sec^2(t)$ for $t \in [0, \frac{\pi}{6}]$.

   (iv) $x = \sin(2t)$, $y = \cos(3t)$ for $t \in [0, \pi]$.

   (v) $x = t^2$, $y = t^3$ for $t \in [0, 2]$.

   (vi) $x = 5\sin(t)$, $y = 3\cos(t)$ for $t \in [0, \frac{\pi}{2}]$.

2. **Find (a) $\frac{dy}{dx}$ and (b) $\frac{d^2y}{dx^2}$ in terms of t:**

   (i) $x = t^2 + 1$, $y = t^3 - 2$.

   (ii) $x = e^t$, $y = e^{-t}$.

   (iii) $x = \cos(t)$, $y = \sin(2t)$.

   (iv) $x = \ln(t)$, $y = t^2$.

   (v) $x = t^3 - 3t$, $y = t^2 + 1$.

(vi) $x = \sqrt{t}$, $y = \ln(t)$.

3. **Determine the arc length of the parametric curve for the given intervals:**

   (i) $x = t$, $y = t^2$, $t \in [0, 3]$.

   (ii) $x = \cos(t)$, $y = \sin(t)$, $t \in [0, \pi]$.

   (iii) $x = e^t$, $y = e^{-t}$, $t \in [0, 1]$.

Challenge Problem

1. A particle's motion is described by $x(t) = t^2 - 4t + 3$ and $y(t) = t^3 - 6t^2 + 11t$. Find the points where the particle changes direction and determine the type of extrema (local max, local min, or neither) for the path of the particle.

Multiple Choice Questions

1. For the parametric equations $x(t) = t^2$ and $y(t) = t^3$, the value of $\frac{dy}{dx}$ at $t = 2$ is:

   a. 6

   b. 12

   c. 4

   d. 3

2. For $x(t) = e^t$ and $y(t) = e^{-t}$, the slope $\frac{dy}{dx}$ at $t = 0$ is:

   a. -1

   b. 1

   c. 0

   d. 2

3. The parametric equations $x(t) = \cos(t)$ and $y(t) = \sin(t)$ represent:

   a. A parabola

   b. A line

   c. A circle

   d. An ellipse

4. The arc length of the parametric curve $x(t) = \sin(t)$ and $y(t) = \cos(t)$ over $t \in [0, 2\pi]$ is:

   a. 2π

   b. π

   c. 1

   d. 4π