AP Calculus BC

Topic: Polar Functions

Instructions

Solve the following problems related to polar functions. Convert coordinates where required and identify graphs using polar equations. Show all your work for full credit.

Practice Problems

1. **Plot each point with the given polar coordinates and find the corresponding rectangular coordinates:**

   (i) $\left(\sqrt{3}, \frac{\pi}{3}\right)$
   (ii) $(2, \pi)$
   (iii) $\left(-3, \frac{2\pi}{3}\right)$
   (iv) $\left(-2, \frac{5\pi}{4}\right)$
   (v) $\left(1, -\frac{\pi}{6}\right)$

2. **Plot each point with the given rectangular coordinates and find two sets of corresponding polar coordinates:**

   (i) $(-2, -\sqrt{2})$
   (ii) $(\sqrt{3}, -1)$
   (iii) $(-1, 0)$
   (iv) $(3, \sqrt{5})$
   (v) $(0, -4)$

3. **Graph the set of points whose polar coordinates satisfy the given equation:**

(i) $r = 2$
(ii) $r = -4$
(iii) $r^2 = 5$
(iv) $\theta = \pi/3$
(v) $r^2 + 2r = 6$

4. **Find an appropriate window and use a graphing calculator to produce the polar curve. Then sketch the complete curve and identify the type of curve by name:**

(i) $r = 1 + \cos\theta$
(ii) $r = 3 - 2\cos\theta$
(iii) $r = 2 - \sin 2\theta$
(iv) $r = 4\cos 2\theta$
(v) $r = \sin\theta$
(vi) $r = 2\cos\theta$
(vii) $r = 1 + 3\sin\theta$
(viii) $r = 5 - 2\sin 2\theta$

5. **Use analytic methods to replace the polar equation by an equivalent Cartesian (rectangular) equation. Then identify or describe the graph without using a grapher:**

(i) $r = 3\cos\theta$
(ii) $r^2 = 16\sin\theta$
(iii) $\theta = \pi/4$
(iv) $r = 4\sin\theta$
(v) $r\cos\theta + r\sin\theta = 2$
(vi) $r^2 = 2r\cos\theta$

6. **Calculate the area of the regions bounded by the given polar curves:**

(i) Inside the cardioid $r = 3 + 2\cos\theta$
(ii) Inside one petal of the rose curve $r = 2\sin 3\theta$
(iii) Shared by the circles $r = 2\cos\theta$ and $r = 2\sin\theta$
(iv) Inside the lemniscate $r^2 = 4\cos 2\theta$

7. **Plot the following polar graphs and label key points. Use a graphing calculator or plotting software if needed:**

(i) $r = 1 + 2\sin\theta$
(ii) $r = 2 - 3\cos\theta$
(iii) $r = 4\sin 2\theta$
(iv) $r^2 = 9\cos 2\theta$

Challenge Problem

A particle moves in the plane such that its position at time t is given in polar coordinates by $r(t) = 2 + \sin t$ and $\theta(t) = \pi/4t$. Find:

(i) The particle's velocity vector at $t = \pi$

(ii) The total distance traveled by the particle for $t \in [0, 2\pi]$