



# AP Calculus BC

## Topic: Power Series

### Instructions

Solve the following problems. Show all your work clearly.

### Practice Problems

1. Determine the interval of convergence for the following power series:

(i)  $\sum_{n=1}^{\infty} \frac{x^n}{n}$

(ii)  $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n+1}$

(iii)  $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$

2. Find the radius of convergence for the given power series:

(i)  $\sum_{n=0}^{\infty} \frac{(2x)^n}{n!}$

(ii)  $\sum_{n=1}^{\infty} \frac{x^{2n}}{n}$

(iii)  $\sum_{n=0}^{\infty} \frac{n! x^n}{3^n}$

3. Expand the following functions into their Maclaurin series:

- (i)  $f(x) = e^x$
- (ii)  $f(x) = \sin(x)$
- (iii)  $f(x) = \ln(1+x)$

4. Use a power series to approximate the following integrals:

- (i)  $\int_0^{0.1} e^{-x^2} dx$  (use the first three terms of the series)
- (ii)  $\int_0^1 \ln(1+x) dx$  (use the first four terms of the series)

5. Determine whether the following series converge or diverge:

- (i)  $\sum_{n=1}^{\infty} \frac{x^n}{n^3}$
- (ii)  $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n}$
- (iii)  $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!} x^n$

## Challenge Problem

1. Prove that the series  $\sum_{n=0}^{\infty} \frac{x^n}{n!}$  converges to  $e^x$  for all  $x \in \mathbb{R}$ .

## Multiple Choice Questions

1. What is the radius of convergence of the series  $\sum_{n=0}^{\infty} \frac{x^n}{n!}$ ?
  - a. 0
  - b. 1
  - c.  $\infty$
  - d. None of the above
2. What is the interval of convergence of the series  $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$ ?
  - a.  $(-1, 1)$
  - b.  $[-1, 1]$
  - c.  $(-\infty, \infty)$
  - d. None of the above
3. Which of the following is the Maclaurin series for  $\sin(x)$ ?

a.  $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$

b.  $\sum_{n=0}^{\infty} \frac{x^n}{n!}$

c.  $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$

d. None of the above

4. What is the sum of the series  $\sum_{n=0}^{\infty} \frac{1}{2^n}$ ?

a. 2

b. 1

c.  $\infty$

d. None of the above

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