



AP Calculus BC

Topic: Radius of Convergence

Instructions

Solve the problems below to practice determining the radius of convergence for various power series. Show your work step by step and justify your answers.

Practice Problems

1. Find the radius of convergence for the following power series:

(i) $\sum_{n=1}^{\infty} \frac{x^n}{n}$

(ii) $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n+1}$

(iii) $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$

(iv) $\sum_{n=0}^{\infty} \frac{(3x)^n}{n!}$

(v) $\sum_{n=1}^{\infty} \frac{nx^n}{2^n}$

2. Determine the interval of convergence for the following power series:

(i) $\sum_{n=0}^{\infty} \frac{(2x)^n}{n!}$

$$\begin{aligned} \text{(ii)} \quad & \sum_{n=1}^{\infty} \frac{x^{2n}}{n} \\ \text{(iii)} \quad & \sum_{n=0}^{\infty} \frac{n!x^n}{3^n} \\ \text{(iv)} \quad & \sum_{n=1}^{\infty} \frac{x^{n+1}}{n^2} \\ \text{(v)} \quad & \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{n!} \end{aligned}$$

3. Use the Ratio Test to determine the radius of convergence of the series

$$\sum_{n=1}^{\infty} \frac{x^n}{n!}.$$

- (i) Find the interval of convergence.
- (ii) Verify the endpoints of the interval.

4. Find the radius of convergence of the following series using the Root Test:

$$\begin{aligned} \text{(i)} \quad & \sum_{n=1}^{\infty} \frac{x^n}{2^n} \\ \text{(ii)} \quad & \sum_{n=1}^{\infty} \frac{n^2 x^n}{3^n} \\ \text{(iii)} \quad & \sum_{n=1}^{\infty} \frac{x^{2n}}{n^3} \\ \text{(iv)} \quad & \sum_{n=0}^{\infty} \frac{(5x)^n}{n^2} \\ \text{(v)} \quad & \sum_{n=1}^{\infty} \frac{x^n}{n!} \end{aligned}$$

5. Prove that the radius of convergence for $\sum_{n=0}^{\infty} \frac{x^n}{n^3}$ is 1.

- (i) Show the steps using the Ratio Test.
- (ii) Verify convergence at the endpoints.

Multiple Choice Questions

1. What is the radius of convergence of the series $\sum_{n=0}^{\infty} \frac{x^n}{n!}$?

- a. 0

- b. 1
- c. ∞
- d. None of the above

2. What is the interval of convergence of the series $\sum_{n=1}^{\infty} \frac{x^n}{n^2}$?

- a. $(-1, 1)$
- b. $[-1, 1]$
- c. $(-\infty, \infty)$
- d. None of the above

3. Which of the following series has a radius of convergence of 2?

a. $\sum_{n=1}^{\infty} \frac{x^n}{2^n}$

b. $\sum_{n=0}^{\infty} \frac{n!x^n}{2^n}$

c. $\sum_{n=1}^{\infty} \frac{x^{2n}}{n!}$

- d. None of the above

4. What is the radius of convergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n}$?

- a. 0
- b. 1
- c. ∞
- d. None of the above

5. The series $\sum_{n=0}^{\infty} \frac{x^n}{n^2}$ converges absolutely for which values of x ?

- a. $|x| < 1$
- b. $|x| \leq 1$
- c. $|x| > 1$
- d. None of the above