



AP Calculus BC

Topic: Relative Rates of Growth

Instructions

Solve the following problems. Show all your work clearly.

Practice Problem

1. Compare the growth rates of the following functions as $x \rightarrow \infty$:

- (i) $f(x) = \ln(x)$ and $g(x) = \sqrt{x}$
- (ii) $f(x) = e^x$ and $g(x) = x^3$
- (iii) $f(x) = x!$ and $g(x) = 2^x$

2. Determine whether the following limits exist. If they do, evaluate them:

- (i) $\lim_{x \rightarrow \infty} \frac{\ln(x)}{x}$
- (ii) $\lim_{x \rightarrow \infty} \frac{x^2}{e^x}$
- (iii) $\lim_{x \rightarrow \infty} \frac{x!}{x^x}$

3. Rank the following functions by their relative rates of growth as $x \rightarrow \infty$:

- (i) $x^2, \ln(x), e^x, x!$
- (ii) $2^x, x^3, \sqrt{x}, e^{2x}$
- (iii) $\ln(x^2), x, \ln(\ln(x))$

4. For the functions $f(x) = x^2$ and $g(x) = e^x$:

- (i) Compute $\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)}$.

(ii) Interpret the result in terms of the relative rates of growth of f and g .

5. Evaluate the following limits using L'Hôpital's Rule to analyze relative growth rates:

(i) $\lim_{x \rightarrow \infty} \frac{e^x}{x^3}$

(ii) $\lim_{x \rightarrow \infty} \frac{\ln(x)}{x^2}$

(iii) $\lim_{x \rightarrow \infty} \frac{x^x}{e^{x^2}}$

Challenge Problems

1. Prove that $\lim_{x \rightarrow \infty} \frac{\ln(x)}{x^p} = 0$ for any $p > 0$.

Multiple Choice Questions

1. Which function grows faster as $x \rightarrow \infty$?

- a. $f(x) = x^2$
- b. $g(x) = 2^x$
- c. They grow at the same rate.
- d. None of the above.

2. What is $\lim_{x \rightarrow \infty} \frac{e^x}{x^x}$?

- a. 0
- b. 1
- c. ∞
- d. Does not exist

3. Which of the following statements is true?

- a. x^3 grows faster than e^x .
- b. $\ln(x)$ grows faster than x^2 .
- c. $x!$ grows faster than 2^x .
- d. None of these.

4. Evaluate $\lim_{x \rightarrow \infty} \frac{\ln(\ln(x))}{\ln(x)}$:

- a. 0
- b. 1
- c. ∞
- d. Does not exist

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