# AP Calculus BC

## Topic: Sequences

**Instructions**

Solve the following problems. Show all your work clearly. Include explanations where necessary.

## Practice Problems

1. **Determine whether the sequence converges or diverges. If it converges, find its limit:**

   (i) $a_n = \frac{1}{n}$

   (ii) $a_n = \frac{2n+3}{5n-1}$

   (iii) $a_n = (-1)^n \cdot \frac{n}{n+1}$

2. **Find the general formula for the $n$th term of the sequence:**

   (i) $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$

   (ii) $2, 4, 8, 16, \ldots$

   (iii) $5, 10, 17, 26, \ldots$

3. **Given the sequence $a_n = \frac{1}{n^2}$, find:**

   (i) The sum of the first 10 terms.

   (ii) Whether the sequence converges or diverges.

4. **Find the first four terms and determine whether the sequence is arithmetic, geometric, or neither:**

   (i) $a_n = 3n + 1$

(ii) $a_n = 2 \cdot 3^n$

(iii) $a_n = \frac{n^2-1}{n+1}$

5. **Evaluate the following limits to determine the behavior of the sequences:**

   (i) $\lim_{n\to\infty} \frac{2n+1}{n-3}$

   (ii) $\lim_{n\to\infty} \frac{(-1)^n}{n}$

   (iii) $\lim_{n\to\infty} \frac{\sin(n)}{n^2}$

6. **A sequence is defined recursively by $a_1 = 2$ and $a_{n+1} = \frac{a_n}{2} + 1$. Find:**

   (i) The first five terms of the sequence.

   (ii) Whether the sequence converges or diverges.

# Challenge Problem

1. A sequence $a_n$ is defined as $a_1 = 1$, and $a_{n+1} = \frac{a_n+2}{a_n+3}$ for $n \geq 1$. Prove that the sequence converges and find its limit.

# Multiple Choice Questions

1. What is the limit of the sequence $a_n = \frac{5n}{n+2}$ as $n \to \infty$?

   a. 5

   b. 2

   c. 0

   d. Diverges

2. Which of the following sequences is geometric?

   a. $1, 4, 9, 16, \ldots$

   b. $2, 6, 18, 54, \ldots$

   c. $5, 10, 15, 20, \ldots$

   d. None of these

3. The sequence $a_n = \frac{n}{n+1}$:

   a. Converges to 1

   b. Diverges

   c. Oscillates

   d. Converges to 0