AP Calculus BC

Topic: Taylor Series

Instructions

Solve the following problems. Show all your work clearly. Include explanations where necessary.

Practice Problems

1. **Find the Taylor series expansion for the following functions about $x = 0$:**

   (i) $f(x) = e^x$

   (ii) $f(x) = \cos(x)$

   (iii) $f(x) = \ln(1 + x)$

2. **Determine the radius of convergence and interval of convergence for the following Taylor series:**

   (i) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$

   (ii) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n}$

   (iii) $\displaystyle\sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$

3. **Approximate the value of the following using the first four terms of the Taylor series:**

   (i) $e^{0.5}$

(ii) $\sin(\frac{\pi}{6})$

(iii) $\ln(1.1)$

4. **Find the Taylor polynomial of degree 3 for the function $f(x) = \sin(x)$ about $x = 0$. Use this polynomial to approximate $\sin(0.2)$.**

5. **Prove that the Taylor series for $f(x) = \frac{1}{1-x}$ about $x = 0$ converges to $f(x)$ for $|x| < 1$.**

Challenge Problem

1. Show that the Taylor series for $f(x) = \arctan(x)$ about $x = 0$ converges to $f(x)$ for $|x| \leq 1$.

Multiple Choice Questions

1. What is the Taylor series expansion for $f(x) = \cos(x)$ about $x = 0$?

   a. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$

   b. $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$

   c. $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$

   d. None of the above

2. The radius of convergence of the Taylor series $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$ is:

   a. 1

   b. ∞

   c. 0

   d. None of the above

3. Which of the following functions has a Taylor series expansion that converges for all $x \in \mathbb{R}$?

   a. $f(x) = \frac{1}{1+x}$

   b. $f(x) = \sin(x)$

   c. $f(x) = \ln(1+x)$

   d. None of the above