AP Calculus BC

Topic: Taylor's Theorem

Instructions

Solve the problems below to practice applying Taylor's Theorem. Clearly justify your steps, and include error bounds wherever applicable.

Practice Problems

1. Find the Taylor polynomial of degree 4 for the following functions about $x = 0$:

   (i) $f(x) = e^x$

   (ii) $f(x) = \sin(x)$

   (iii) $f(x) = \ln(1 + x)$

2. Use Taylor's Theorem to find an upper bound for the error when approximating $\sin(0.1)$ using the third-degree Taylor polynomial about $x = 0$.

3. Approximate $e^{0.5}$ using the second-degree Taylor polynomial for $f(x) = e^x$ about $x = 0$. Find the error bound using Taylor's Theorem.

4. Determine the degree of the Taylor polynomial required to approximate $\cos(0.2)$ with an error less than 0.001 using Taylor's Theorem.

5. Prove that the Taylor series for $f(x) = \frac{1}{1-x}$ about $x = 0$ converges to $f(x)$ for $|x| < 1$ using Taylor's Theorem.

Challenge Problem

1. Show that the Taylor series for $f(x) = e^x$ about $x = 0$ converges to $f(x)$ for all $x \in \mathbb{R}$ and provide the error bound for $|x| \leq 2$ when using the fourth-degree Taylor polynomial.

Multiple Choice Questions

1. What is the remainder term in Taylor's Theorem?

   a. $R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!}x^{n+1}$, where c is between 0 and x

   b. $R_n(x) = \frac{f^{(n)}(c)}{n!}x^{n}$, where c is between 0 and x

   c. $R_n(x) = \frac{f'(c)}{n!}x^{n+1}$, where c is between 0 and x

   d. None of the above

2. What is the Taylor polynomial of degree 3 for $f(x) = \cos(x)$ about $x = 0$?

   a. $1 - \frac{x^2}{2} + \frac{x^4}{24}$

   b. $1 - \frac{x^2}{2} + \frac{x^3}{6}$

   c. $1 - \frac{x^2}{2}$

   d. None of the above

3. Which of the following is a correct Taylor polynomial of degree 2 for $f(x) = \ln(1+x)$ about $x = 0$?

   a. $x - \frac{x^2}{2}$

   b. $1 + x - \frac{x^2}{2}$

   c. $x + \frac{x^2}{2}$

   d. None of the above

4. What is the error bound for approximating e^x using the third-degree Taylor polynomial for $|x| \leq 1$?

   a. $\frac{e}{6}$

   b. $\frac{1}{6}$

   c. $\frac{e}{24}$

   d. None of the above