



AP Calculus BC

Topic: Testing Convergence at Endpoints

Instructions

Solve each question by determining whether the series converges or diverges at the given endpoints. Justify your answers using proper tests such as the Integral Test, Comparison Test, or Alternating Series Test.

Practice Problems

1. Determine whether the series converges or diverges at the endpoints:

(i) $\sum_{n=1}^{\infty} \frac{x^n}{n!}$ at $x = -1$ and $x = 1$.

(ii) $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n}$ at $x = -2$ and $x = 2$.

(iii) $\sum_{n=1}^{\infty} \frac{n^2 x^n}{3^n}$ at $x = -3$ and $x = 3$.

2. Using the Limit Comparison Test, verify the behavior of the series:

(i) $\sum_{n=1}^{\infty} \frac{x^n}{n^3}$ at $x = -1$ and $x = 1$.

(ii) $\sum_{n=1}^{\infty} \frac{x^{2n}}{\sqrt{n}}$ at $x = -2$ and $x = 2$.

(iii) $\sum_{n=1}^{\infty} \frac{(2x)^n}{n^2}$ at $x = -1$ and $x = 1$.

3. For the following power series, analyze the endpoints for absolute or conditional convergence:

(i) $\sum_{n=1}^{\infty} \frac{x^{n+1}}{n^2}$ at $x = -1$ and $x = 1$.

(ii) $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n^3}$ at $x = -2$ and $x = 2$.

(iii) $\sum_{n=0}^{\infty} \frac{(x^2)^n}{n!}$ at $x = -1$ and $x = 1$.

4. Prove convergence or divergence using the Alternating Series Test:

(i) $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{\sqrt{n}}$ at $x = -1$ and $x = 1$.

(ii) $\sum_{n=1}^{\infty} \frac{x^n}{2^n}$ at $x = -2$ and $x = 2$.

(iii) $\sum_{n=1}^{\infty} \frac{(-1)^n x^{n+1}}{n!}$ at $x = -3$ and $x = 3$.

Multiple Choice Questions

- What is the convergence status of $\sum_{n=1}^{\infty} \frac{x^n}{n}$ at $x = -1$?
 - Absolutely Convergent
 - Conditionally Convergent
 - Divergent
 - None of the above
- For $\sum_{n=1}^{\infty} \frac{n^2 x^n}{3^n}$, what happens at $x = 3$?
 - Absolutely Convergent
 - Conditionally Convergent
 - Divergent
 - None of the above
- Does the series $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n^2}$ converge absolutely at $x = 2$?
 - Yes
 - No
 - Converges conditionally

- d. None of the above
4. For $\sum_{n=0}^{\infty} \frac{nx^n}{3^n}$, what happens at $x = 3$?
- a. Absolutely Convergent
 - b. Conditionally Convergent
 - c. Divergent
 - d. None of the above
5. The series $\sum_{n=1}^{\infty} \frac{x^n}{n^3}$ is:
- a. Absolutely Convergent at $x = -1$
 - b. Divergent at $x = 1$
 - c. Conditionally Convergent at $x = -1$
 - d. None of the above

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