AP Calculus BC

Topic: Vectors in the Plane

Instructions

Solve the following problems involving vectors in the plane. Use appropriate vector operations and include diagrams where required. Show all your work for full credit.

Practice Problems

1. **Find the magnitude and direction of the following vectors:**
   (i) $\mathbf{v} = \langle 3, 4 \rangle$
   (ii) $\mathbf{w} = \langle -2, 5 \rangle$
   (iii) $\mathbf{u} = \langle 7, -24 \rangle$
2. **Perform the following vector operations:**
   (i) $2\mathbf{v} + 3\mathbf{w}$, where $\mathbf{v} = \langle 1, 2 \rangle$ and $\mathbf{w} = \langle 3, -1 \rangle$.
   (ii) $\mathbf{u} - \mathbf{v}$, where $\mathbf{u} = \langle 5, 6 \rangle$ and $\mathbf{v} = \langle -2, 3 \rangle$.
   (iii) $\mathbf{v} \cdot \mathbf{w}$ (dot product), where $\mathbf{v} = \langle 4, -3 \rangle$ and $\mathbf{w} = \langle -1, 2 \rangle$.
3. **Determine whether the following vectors are orthogonal:**
   (i) $\mathbf{v} = \langle 1, -1 \rangle$, $\mathbf{w} = \langle -1, -1 \rangle$
   (ii) $\mathbf{u} = \langle 3, 4 \rangle$, $\mathbf{w} = \langle -8, 6 \rangle$
4. **Find the angle between the vectors $\mathbf{v}$ and $\mathbf{w}$:**
   (i) $\mathbf{v} = \langle 1, 2 \rangle$, $\mathbf{w} = \langle 3, 4 \rangle$

(ii) $\mathbf{v} = \langle -2, -3 \rangle$, $\mathbf{w} = \langle 4, -5 \rangle$

5. **A particle moves in the plane so that its position at any time t is given parametrically by $x = 2\sin t$ and $y = 3\cos t$, for $0 \leq t \leq 2\pi$.**

   (i) Find the velocity vector for the particle.

   (ii) At what values of t is the particle at rest?

   (iii) Sketch the path of the particle.

   (iv) Write an equation for the path of the particle that relates x and y directly.

6. **The position of a particle at any time t is given by $x(t) = e^t \cos t$ and $y(t) = e^t \sin t$, for $0 \leq t \leq \pi$.**

   (i) Find the slope of the path at time $t = \frac{\pi}{4}$.

   (ii) Compute the speed of the particle at $t = 1$.

   (iii) Determine the total distance traveled by the particle from $t = 0$ to $t = \pi$.

7. **The velocity vector of a particle is given by $\mathbf{v}(t) = \langle t^2, 2t \rangle$. Assume the particle starts at $(1, 0)$ when $t = 0$.**

   (i) Find the position vector $\mathbf{r}(t)$.

   (ii) Determine the position of the particle at $t = 3$.

   (iii) Find the total distance traveled by the particle from $t = 0$ to $t = 3$.

8. **A particle moves along a curve in the xy-plane such that its position at time $t \geq 0$ is given by $x(t) = t^2 - 4$ and $y(t) = 3t^3$.**

   (i) Find $\frac{dy}{dx}$ as a function of t.

   (ii) At $t = 2$, compute the slope of the tangent line to the curve.

   (iii) Determine the equation of the tangent line at $t = 2$.

Multiple Choice Questions

1. What is the magnitude of the vector $\mathbf{v} = \langle -3, 4 \rangle$?

   a. 5

   b. 7

   c. $\sqrt{25}$

   d. $\sqrt{29}$

2. If $\mathbf{v} = \langle 2, -1 \rangle$ and $\mathbf{w} = \langle 1, 3 \rangle$, what is $\mathbf{v} \cdot \mathbf{w}$?

   a. -1

   b. 1

   c. 5

   d. 7

3. Which of the following pairs of vectors are orthogonal?

   a. $\mathbf{v} = \langle 1, 2 \rangle$, $\mathbf{w} = \langle -2, 1 \rangle$

   b. $\mathbf{v} = \langle 3, 4 \rangle$, $\mathbf{w} = \langle 4, -3 \rangle$

   c. Both (a) and (b)

   d. None of the above

4. The angle between $\mathbf{v} = \langle 1, 0 \rangle$ and $\mathbf{w} = \langle 0, 1 \rangle$ is:

   a. $0°$

   b. $45°$

   c. $90°$

   d. $180°$

5. A vector $\mathbf{u} = \langle x, y \rangle$ satisfies $|\mathbf{u}| = 5$ and $x = 3$. What is y?

   a. ± 4

   b. ± 3

   c. ± 2

   d. ± 5