AP Calculus BC

Topic: Volumes

Instructions

Solve the following problems. Show all your work clearly and include units in your answers where appropriate.

Practice Problems

1. **Find the volume of the solid generated by revolving the region bounded by the given functions about the x-axis:**
   i. $y = x^2$, $y = 0$, $x = 2$.
   ii. $y = \sqrt{x}$, $x = 1$, $x = 4$.
   iii. $y = 3 - x$, $y = 0$, $x = 0$.
2. **Find the volume of the solid generated by revolving the region bounded by the given functions about the y-axis:**
   i. $x = \sqrt{y}$, $y = 4$, $y = 0$.
   ii. $x = 2y$, $x = 0$, $y = 3$.
3. **Compute the volumes of the following solids using known cross-sections:**
   i. A solid with a base bounded by $y = x^2$ and $y = 4 - x^2$, where the cross-sections perpendicular to the x-axis are squares.
   ii. A solid with a base bounded by $x = y^2$ and $x = 4$, where the cross-sections perpendicular to the y-axis are semicircles.

Multiple Choice Questions

1. The volume of the solid obtained by revolving $y = x^3$ on $[0, 1]$ about the x-axis is:

   a. $\frac{\pi}{4}$
   b. $\frac{\pi}{5}$
   c. $\frac{\pi}{6}$
   d. $\frac{\pi}{3}$

2. The volume of the solid obtained by revolving $y = 2x - x^2$ on $[0, 2]$ about the x-axis is:

   a. π
   b. 2π
   c. $\frac{8\pi}{3}$
   d. $\frac{4\pi}{3}$

3. The volume of the solid obtained by revolving $y = \ln(x)$ on $[1, e]$ about the x-axis is:

   a. $\pi(e - 2)$
   b. $\pi(e - 1)$
   c. $\pi(1 - e)$
   d. $\pi(2 - e)$

Challenge Problem

1. Find the volume of the solid generated by revolving the region bounded by $y = \sin(x)$, $y = 0$, $x = 0$, and $x = \pi$ about the x-axis.