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DERIVATIVE OF A FUNCTION

AP Calculus AB Practice Worksheet

Learning Goal: Understand the derivative as the slope of a tangent line and an instantaneous rate of change, compute derivatives using the limit definition, and connect the graphs of f and f' .

1. Understanding the Derivative

Answer each question and show your reasoning.

1. For

$$f(x) = x^2,$$

find the average rate of change of f on the interval $[1, 3]$. What does this number represent on the graph of f ?

Work:

2. In your own words, explain the difference between the slope of a *secant line* and the slope of a *tangent line*.

Work:

2. Estimate the Derivative from a Table

For

$$f(x) = x^2,$$

complete the table of difference quotients $\frac{f(3+h) - f(3)}{h}$.

h	-0.1	-0.01	-0.001	0.001	0.01
$\frac{f(3+h) - f(3)}{h}$					

Estimate: $f'(3) =$ _____

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3. Definition of the Derivative

The derivative of f with respect to x is the function f' given by:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$
$$f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} \quad (\text{alternate form at } x = a)$$

Common notations for the derivative of $y = f(x)$:

$$f'(x) \quad y' \quad \frac{dy}{dx} \quad \frac{d}{dx}f(x)$$

4. Derivative at a Point

Use the definition of the derivative to find $f'(a)$. Show every step.

1.

$$f(x) = 3x - 5, \quad a = 4$$

2.

$$f(x) = x^2 + 1, \quad a = 2$$

3.

$$f(x) = 2x^2 - x, \quad a = -1$$

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5. Find the Derivative Using the Definition

Use $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ to find $f'(x)$.

1.

$$f(x) = x^2 - 4x$$

2.

$$f(x) = x^3$$

3.

$$f(x) = \frac{1}{x}$$

4.

$$f(x) = \sqrt{x} \quad (\text{Hint: multiply by the conjugate.})$$

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6. Equations of Tangent Lines

The tangent line to $y = f(x)$ at $x = a$ has slope $f'(a)$ and passes through $(a, f(a))$:

$$y - f(a) = f'(a)(x - a)$$

Write an equation of the tangent line at the given point.

1.

$$f(x) = x^2, \quad x = 3$$

2.

$$f(x) = \frac{1}{x}, \quad x = 2$$

3.

$$f(x) = \sqrt{x}, \quad x = 4$$

7. Using Derivative Notation

Evaluate each expression. Use your results from Section 5.

1. If $y = x^3$, find $\left. \frac{dy}{dx} \right|_{x=-2}$. _____

2. Find $\frac{d}{dx}(x^2 - 4x)$ at $x = 5$. _____

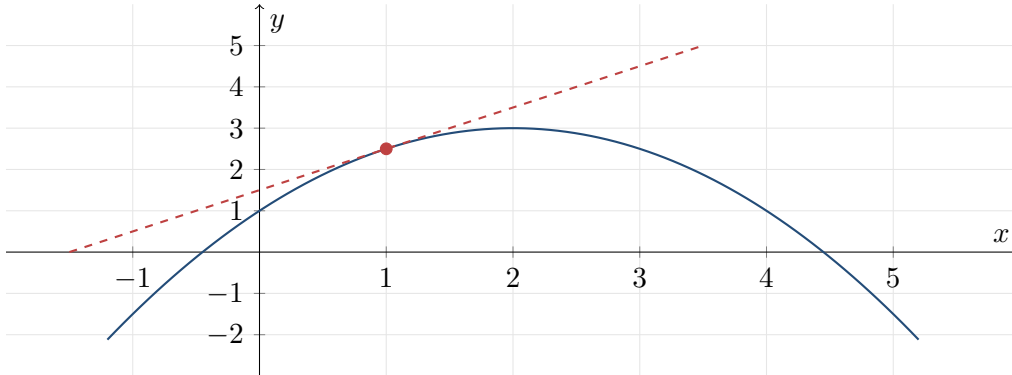
3. At what x -value does $f(x) = x^2 - 4x$ have a horizontal tangent line? _____

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8. Read the Slope from the Graph

The graph of $y = f(x)$ is shown below, together with the tangent line (dashed) at the point $(1, 2.5)$.



Answer the following:

1. Use the tangent line to find $f'(1)$.
2. $f'(2) =$
3. Is $f'(4)$ positive, negative, or zero? Explain.

4. On what interval is $f'(x) > 0$?
5. On what interval is $f'(x) < 0$?

9. Relationships Between the Graphs of f and f'

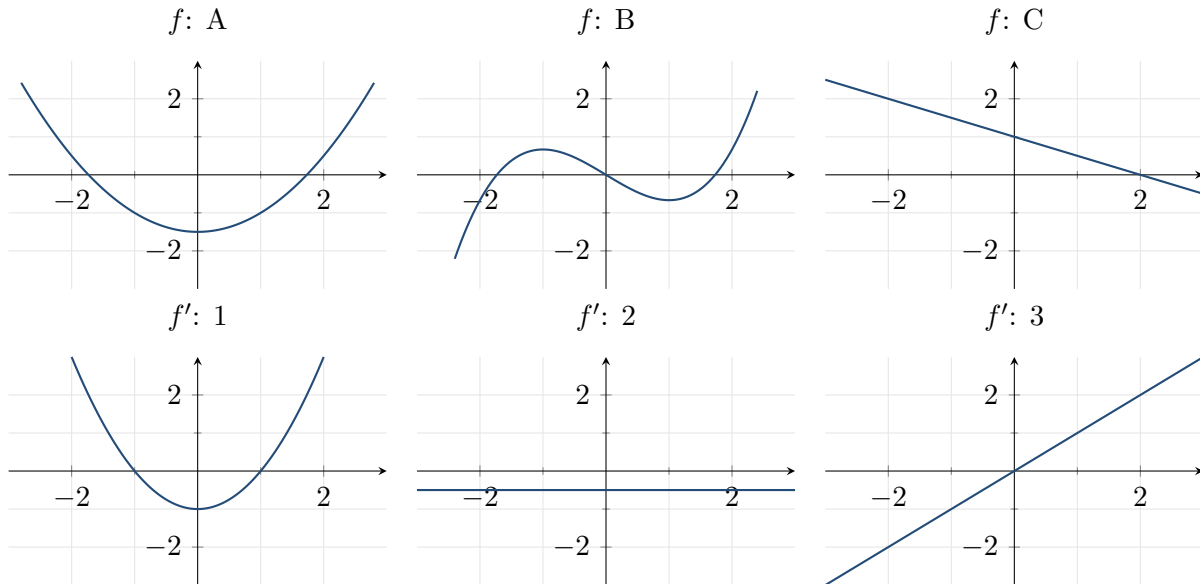
$f'(x) > 0 \implies f$ is increasing
$f'(x) < 0 \implies f$ is decreasing
$f'(x) = 0 \implies f$ has a horizontal tangent line

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10. Match Each Function with Its Derivative

The top row shows graphs of f . The bottom row shows graphs of f' . Match each graph of f with the graph of its derivative.



1. Graph A matches:
2. Graph B matches:
3. Graph C matches:

11. Explain Your Thinking

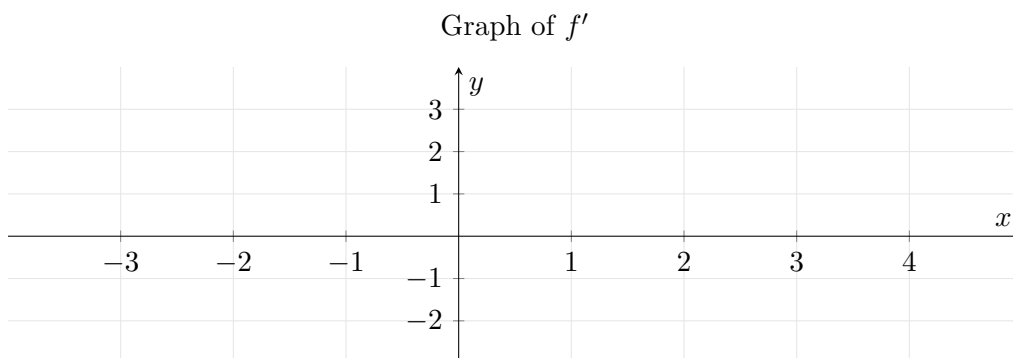
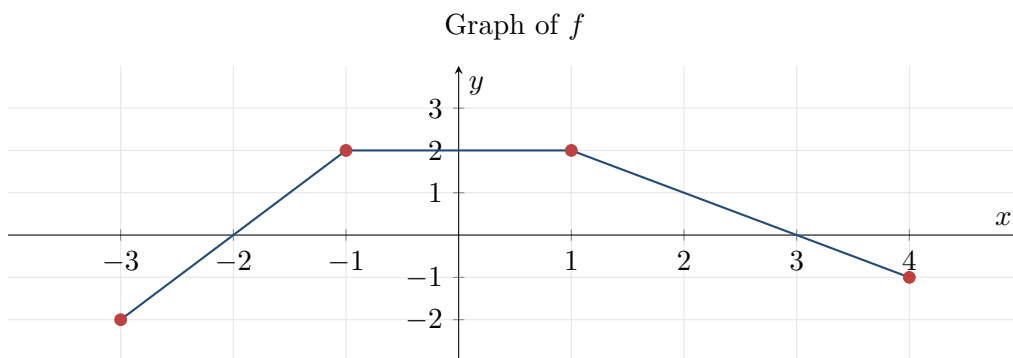
For Graph B, explain how you can locate the x -intercepts of f' by looking only at the graph of f .

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12. Sketch the Graph of the Derivative

The graph of f is made of line segments. Use it to sketch the graph of f' on the blank grid.



1. At which x -values in $(-3, 4)$ is f' undefined? Why?

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13. Graphing the Derivative from Data

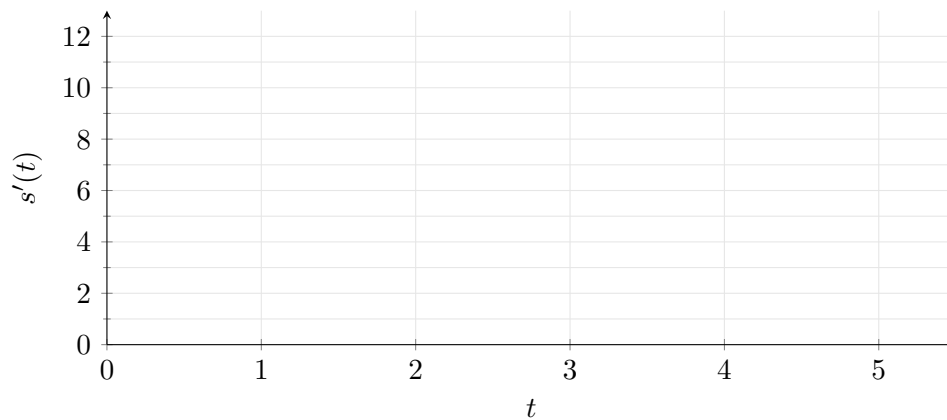
A cart moves along a straight track. Its position $s(t)$, in meters, is recorded every second.

t (sec)	0	1	2	3	4	5
$s(t)$ (m)	0	3	8	15	24	35

1. Complete the table of average velocities $\frac{\Delta s}{\Delta t}$ over each 1-second interval.

Interval	[0, 1]	[1, 2]	[2, 3]	[3, 4]	[4, 5]
Midpoint t	0.5	1.5	2.5	3.5	4.5
$\frac{\Delta s}{\Delta t}$ (m/s)					

2. Plot the points (midpoint t , $\frac{\Delta s}{\Delta t}$) on the grid below and connect them. This is an estimate of the graph of $s'(t)$.



3. Describe the shape of the graph of $s'(t)$. Estimate the velocity of the cart at $t = 3$.

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14. One-Sided Derivatives

At an endpoint or a corner, we look at each side separately:

$$\begin{array}{l} \text{Right-hand derivative: } \lim_{h \rightarrow 0^+} \frac{f(a+h) - f(a)}{h} \\ \text{Left-hand derivative: } \lim_{h \rightarrow 0^-} \frac{f(a+h) - f(a)}{h} \end{array}$$

$f'(a)$ exists only when both one-sided derivatives exist and are equal.

1. For

$$f(x) = |x - 2|,$$

find the left-hand and right-hand derivatives at $x = 2$. Does $f'(2)$ exist?

2. For

$$g(x) = \begin{cases} x^2, & x \leq 1 \\ 2x - 1, & x > 1 \end{cases}$$

find the left-hand and right-hand derivatives at $x = 1$. Is g differentiable at $x = 1$?

15. Explain Your Thinking

The function $f(x) = |x|$ is continuous at $x = 0$. Explain why it is *not* differentiable at $x = 0$.

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Mixed Review

16. Mixed Practice

Show your work clearly.

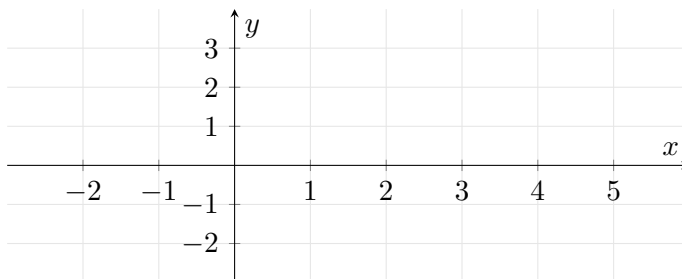
1. Use the definition to find
- $f'(x)$
- for
- $f(x) = 2x^2 - 3$
- .

2. Recognize the limit as a derivative, then evaluate:
- $\lim_{h \rightarrow 0} \frac{(3+h)^2 - 9}{h}$
- .

17. Challenge ProblemFind the values of a and b that make $f(x) = \begin{cases} x^2, & x \leq 1 \\ ax + b, & x > 1 \end{cases}$ differentiable at $x = 1$.**18. Graphing Challenge**

Sketch a continuous function that satisfies all of the following:

- $f(0) = 2$ and $f'(0) = 0$
- $f'(x) > 0$ for $x < 0$
- $f'(x) < 0$ for $0 < x < 3$
- $f'(3) = 0$ and $f'(x) > 0$ for $x > 3$

**Reflection:** What is one thing you understand better about derivatives after completing this worksheet?