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DERIVATIVES OF TRIGONOMETRIC FUNCTIONS

AP Calculus AB Practice Worksheet

Learning Goal: Derive and apply the derivatives of the six trigonometric functions, combine them with the product, quotient, and chain rules, find higher-order derivatives, and use trigonometric derivatives to model motion.

1. Two Special Limits

Use a calculator in **radian mode**. Round to four decimal places.

h	-0.1	-0.01	-0.001	0.001	0.01	0.1
$\frac{\sin h}{h}$						
$\frac{\cos h - 1}{h}$						

$$\lim_{h \rightarrow 0} \frac{\sin h}{h} = \underline{\hspace{2cm}} \qquad \lim_{h \rightarrow 0} \frac{\cos h - 1}{h} = \underline{\hspace{2cm}}$$

2. Unit Circle Review

Give exact values. You will need these throughout the worksheet.

1. $\sin\left(\frac{\pi}{6}\right) = \underline{\hspace{2cm}}$ $\cos\left(\frac{\pi}{3}\right) = \underline{\hspace{2cm}}$ $\tan\left(\frac{\pi}{4}\right) = \underline{\hspace{2cm}}$

2. $\sec\left(\frac{\pi}{3}\right) = \underline{\hspace{2cm}}$ $\csc\left(\frac{\pi}{6}\right) = \underline{\hspace{2cm}}$ $\cot\left(\frac{\pi}{4}\right) = \underline{\hspace{2cm}}$

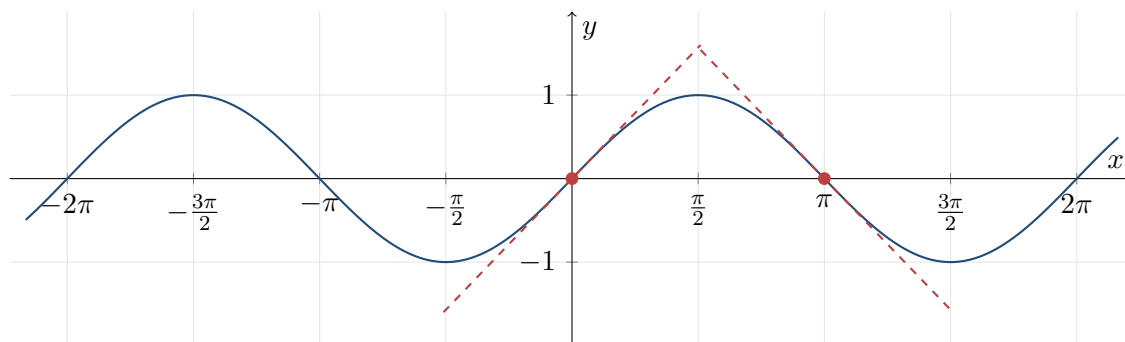
3. $\cos(\pi) = \underline{\hspace{2cm}}$ $\sin\left(\frac{3\pi}{2}\right) = \underline{\hspace{2cm}}$ $\sec^2\left(\frac{\pi}{4}\right) = \underline{\hspace{2cm}}$

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3. Discover the Derivative of $\sin x$ from Its Graph

The graph of $y = \sin x$ is shown with dashed tangent lines at $x = 0$ and $x = \pi$.



1. Use the graph to estimate the slope of $y = \sin x$ at each x -value. Then compare with $\cos x$.

x	$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π
Slope of $\sin x$					
$\cos x$					

2. What function appears to give the slope of $y = \sin x$ at every point?

3. Where on the graph of $y = \sin x$ are the tangent lines horizontal? What is true about $\cos x$ at those points?

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4. Prove the Derivative of $\sin x$

Use the definition of the derivative and the identity $\sin(A + B) = \sin A \cos B + \cos A \sin B$. Fill in each blank.

$$\begin{aligned}\frac{d}{dx}(\sin x) &= \lim_{h \rightarrow 0} \frac{\sin(x + h) - \sin x}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sin(x + h) - \sin x}{h} \\ &= \lim_{h \rightarrow 0} \left[\sin x \cdot \frac{\cos h - 1}{h} + \cos x \cdot \frac{\sin h}{h} \right] \\ &= \sin x \cdot (\text{_____}) + \cos x \cdot (\text{_____}) \\ &= \text{_____}\end{aligned}$$

Which two limits from Section 1 did you use?

5. Your Turn: Prove the Derivative of $\cos x$

Use the identity $\cos(A + B) = \cos A \cos B - \sin A \sin B$ and the definition of the derivative to show that

$$\frac{d}{dx}(\cos x) = -\sin x.$$

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6. Derivative Rules

Let u be a differentiable function of x .

$$\frac{d}{dx}(\sin u) = \cos u \frac{du}{dx}$$

$$\frac{d}{dx}(\cos u) = -\sin u \frac{du}{dx}$$

$$\frac{d}{dx}(\tan u) = \sec^2 u \frac{du}{dx}$$

$$\frac{d}{dx}(\cot u) = -\csc^2 u \frac{du}{dx}$$

$$\frac{d}{dx}(\sec u) = \sec u \tan u \frac{du}{dx}$$

$$\frac{d}{dx}(\csc u) = -\csc u \cot u \frac{du}{dx}$$

Pattern: The derivative of every *co*-function ($\cos$, $\cot$, $\csc$) has a negative sign.

7. Derive the Remaining Rules

Use $\tan x = \frac{\sin x}{\cos x}$, $\sec x = \frac{1}{\cos x}$, and the quotient rule.

1. Show that $\frac{d}{dx}(\tan x) = \sec^2 x$.

2. Show that $\frac{d}{dx}(\sec x) = \sec x \tan x$.

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8. Basic Trigonometric DerivativesFind $\frac{dy}{dx}$.

1.

$$y = 3 \sin x - 4 \cos x$$

2.

$$y = 5 \tan x + 2 \sec x$$

3.

$$y = x^2 - \cot x + \csc x$$

4.

$$y = \sqrt{x} + 7 \cos x - \pi^2$$

5.

$$y = \frac{\sin x}{4} + \frac{2}{x}$$

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9. Product and Quotient Rules with Trig

Find $\frac{dy}{dx}$. Simplify where possible.

1.

$$y = x^2 \sin x$$

2.

$$y = \sin x \cos x$$

3.

$$y = \frac{\sin x}{x}$$

4.

$$y = \frac{1 + \cos x}{1 - \cos x}$$

5.

$$y = x \tan x - \sec x$$

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10. Chain Rule with TrigFind $\frac{dy}{dx}$.

1. $y = \sin(3x)$

2. $y = \cos(x^2 + 1)$

3. $y = \tan(5x - 2)$

4. $y = \sin^2 x$ (Hint: $\sin^2 x = (\sin x)^2$)

5. $y = \sec(\sqrt{x})$

6. $y = \cos^3(2x)$

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11. Combine the Rules

Find $\frac{dy}{dx}$. You will need more than one rule.

1.

$$y = x^3 \cos(2x)$$

2.

$$y = \sin(\cos x)$$

3.

$$y = \frac{\tan(3x)}{x^2 + 1}$$

4.

$$y = \sqrt{1 + \sin^2 x}$$

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12. Evaluate the Derivative at a Point

Find the exact value.

1. If $f(x) = \sin x$, find $f'\left(\frac{\pi}{3}\right)$.

2. If $g(x) = x \cos x$, find $g'(\pi)$.

3. If $h(x) = \tan x$, find $h'\left(\frac{\pi}{4}\right)$.

4. If $k(x) = \sec x$, find $k'\left(\frac{\pi}{6}\right)$.

5. If $m(x) = \sin(2x)$, find $m'\left(\frac{\pi}{3}\right)$.

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13. Equations of Tangent and Normal LinesWrite an equation of the tangent line at the given x -value.

1.

$$y = \sin x, \quad x = \frac{\pi}{6}$$

2.

$$y = \tan x, \quad x = \frac{\pi}{4}$$

3.

$$y = \cos(2x), \quad x = \frac{\pi}{4}$$

4. Write an equation of the *normal* line to

$$y = \sec x$$

at $x = \frac{\pi}{3}$.

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14. Horizontal Tangent Lines

Find every x -value in $[0, 2\pi)$ where the graph has a horizontal tangent line.

1.

$$y = x + 2 \sin x$$

2.

$$y = \sin x + \cos x$$

3.

$$y = 2 \cos x + \sin^2 x$$

15. Explain Your Thinking

The graph of $y = x + \sin x$ never decreases. Use its derivative to explain why.

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16. Higher-Order Derivatives

1. Find the first four derivatives of $y = \sin x$. What pattern do you see?

y	y'	y''	y'''	$y^{(4)}$
$\sin x$				

2. Use the pattern to find

$$\frac{d^{27}}{dx^{27}}(\sin x).$$

3. Find y'' for

$$y = \cos(3x).$$

4. Find y'' for

$$y = \tan x.$$

5. Show that $y = A \sin x + B \cos x$ satisfies

$$y'' + y = 0$$

for any constants A and B .

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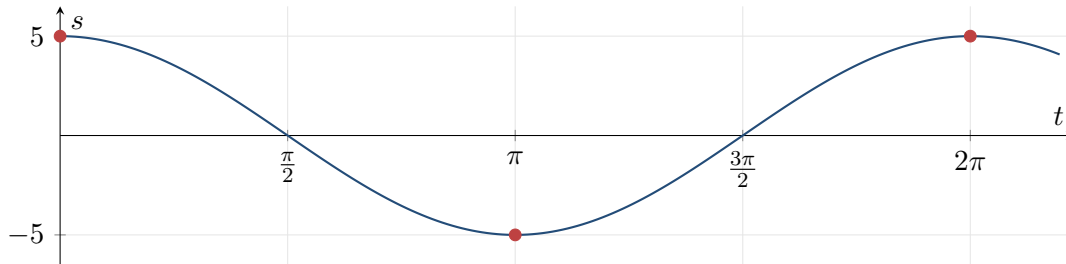
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17. Simple Harmonic Motion

A weight hanging from a spring bobs up and down. Its position, in centimeters, is

$$s(t) = 5 \cos t,$$

where t is in seconds and $s > 0$ means above the rest position.



1. Find the velocity $v(t)$ and acceleration $a(t)$.

2. Find the position, velocity, and acceleration at $t = \frac{\pi}{3}$. Is the weight moving up or down at that moment?

3. When is the weight momentarily at rest? Where is it at those times?

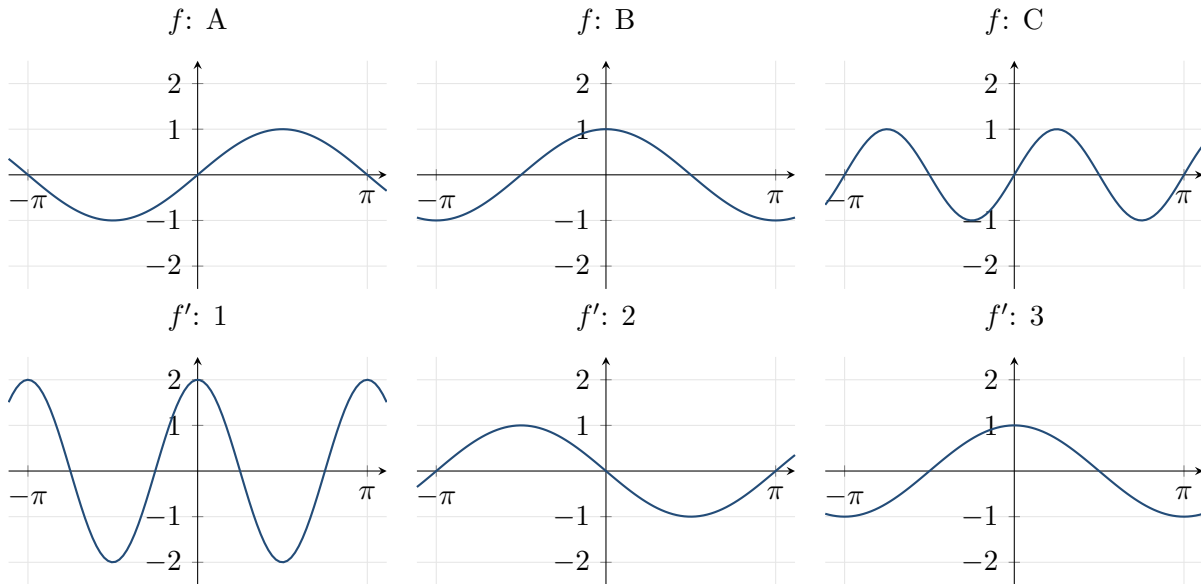
4. When is the speed $|v(t)|$ greatest? Where is the weight at those times?

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18. Match Each Function with Its Derivative

The top row shows graphs of f . The bottom row shows graphs of f' . Match each graph of f with the graph of its derivative.



1. Graph A matches:
2. Graph B matches:
3. Graph C matches:

19. Explain Your Thinking

Graph C oscillates twice as fast as Graph A. Explain, using the chain rule, why its derivative has a maximum value of 2 instead of 1.

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TRIGONOMETRIC DERIVATIVES

Mixed Review

20. Mixed Practice

Show your work clearly.

1. Find
- $\frac{dy}{dx}$
- for
- $y = \csc x \cot x$
- .

2. Find
- $\frac{dy}{dx}$
- for
- $y = \frac{\cos x}{1 + \sin x}$
- .

3. Find
- $\frac{dy}{dx}$
- for
- $y = \tan^2(4x)$
- .

4. Evaluate
- $\lim_{h \rightarrow 0} \frac{\sin(\frac{\pi}{6} + h) - \frac{1}{2}}{h}$
- by recognizing it as a derivative.

21. Challenge Problem

Find the values of a and b that make $f(x) = \begin{cases} a \sin x + b, & x < 0 \\ x^2 + 3x + 2, & x \geq 0 \end{cases}$ differentiable at $x = 0$.

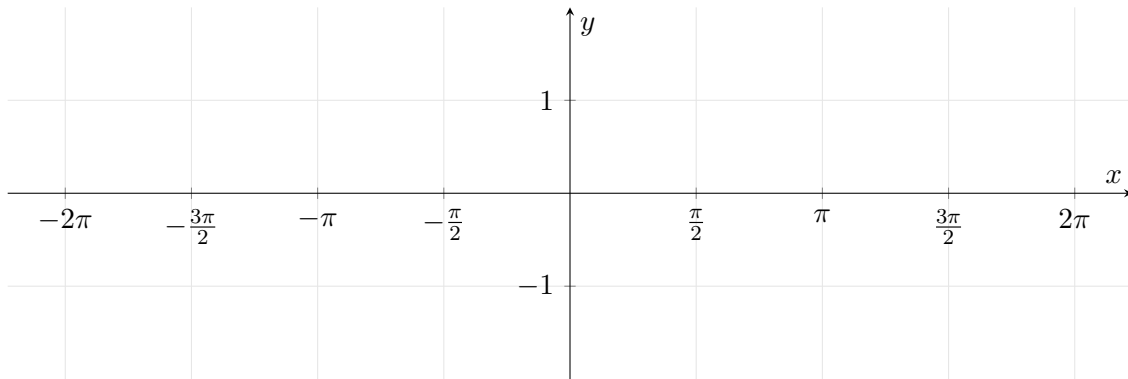
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22. Graphing Challenge

On the grid below:

- Sketch $f(x) = \cos x$ on $[-2\pi, 2\pi]$.
- Mark every point where the tangent line to f is horizontal.
- On the same grid, sketch $f'(x)$ using a different color or a dashed curve.



1. At the points where f has a horizontal tangent, what is the value of f' ?

2. On which intervals is $f'(x) < 0$? What is f doing on those intervals?

Reflection: What is one thing you understand better about derivatives of trigonometric functions after completing this worksheet?
