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DIFFERENTIABILITY

AP Calculus AB Practice Worksheet

Learning Goal: Determine where a function is differentiable, recognize corners, cusps, vertical tangents, and discontinuities, use one-sided derivatives, and understand the relationship between differentiability and continuity.

1. Understanding Differentiability

Answer each question in complete sentences.

1. What does it mean for a function f to be *differentiable* at $x = a$?

Answer:

2. What does it mean for f to be differentiable on an open interval (a, b) ?

Answer:

2. One-Sided Difference Quotients from a Table

Let $f(x) = |x^2 - 4|$. Complete the table of difference quotients at $x = 2$.

h	-0.1	-0.01	-0.001	0.001	0.01	0.1
$\frac{f(2+h) - f(2)}{h}$						

Left-hand derivative $\approx$ _____ Right-hand derivative $\approx$ _____

Is f differentiable at $x = 2$? Explain.

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3. How a Derivative Can Fail to Exist

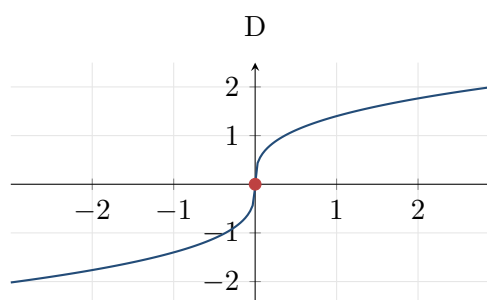
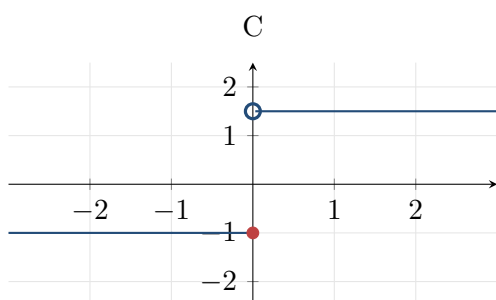
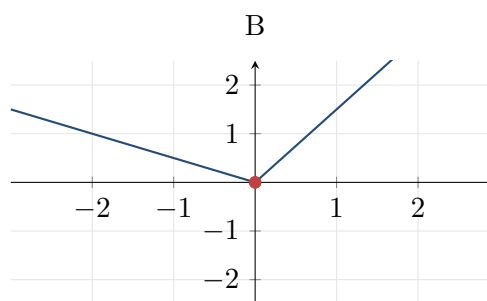
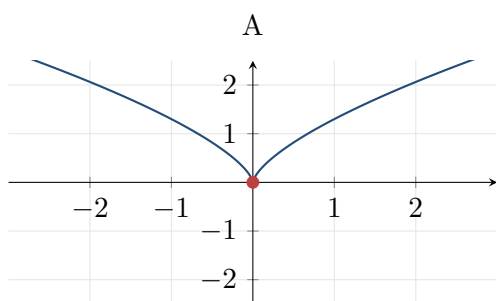
A function fails to be differentiable at $x = a$ when the graph has:

- | | |
|-----------------------|---|
| 1. A corner | The one-sided derivatives exist but are <i>different</i> . |
| 2. A cusp | The slopes approach $+\infty$ on one side and $-\infty$ on the other. |
| 3. A vertical tangent | The slopes approach $+\infty$ (or $-\infty$) from <i>both</i> sides. |
| 4. A discontinuity | A jump, hole, or vertical asymptote at $x = a$. |

4. Classify the Failure

Each graph fails to be differentiable at $x = 0$. Classify each one as:

Corner	Cusp	Vertical Tangent	Discontinuity
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1. Graph A:

Graph B:

2. Graph C:

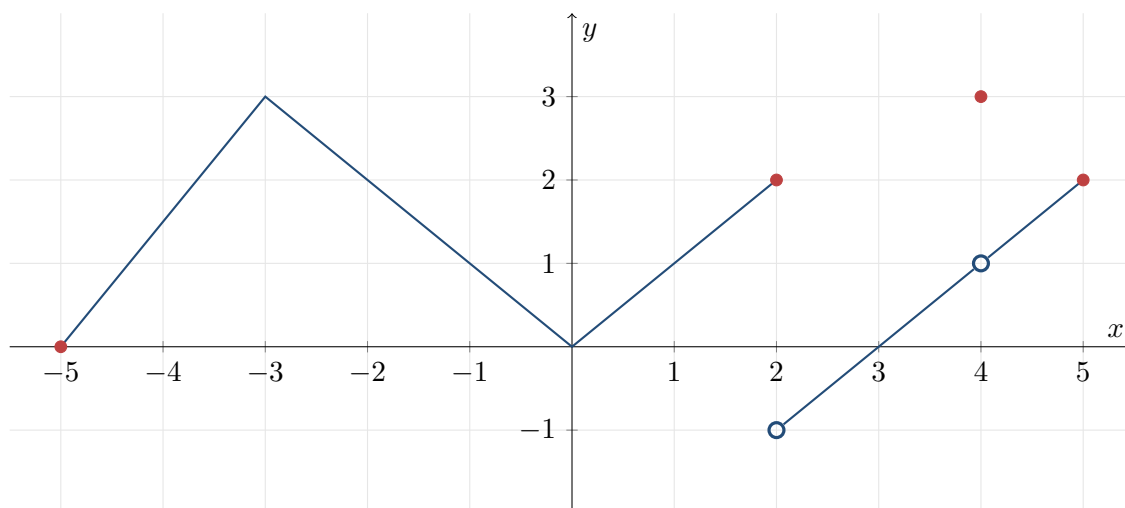
Graph D:

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5. Locate Every Point of Non-Differentiability

The graph of f on $[-5, 5]$ is shown below.



Complete the table for every x -value in $(-5, 5)$ where f is *not* differentiable.

x -value	Type	Reason

- At which of these x -values is f continuous but not differentiable? _____
- Find $f'(-4)$, $f'(-1)$, and $f'(3)$.

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6. Differentiability Implies Continuity

If f is differentiable at $x = a$, then f is continuous at $x = a$.

If f is *not* continuous at $x = a$, then f is *not* differentiable at $x = a$.

The converse is **false**: a function can be continuous at $x = a$ without being differentiable there.

7. True or False?

Write **True** or **False**. If false, give a counterexample (a formula or a quick sketch).

1. If f is continuous at $x = a$, then f is differentiable at $x = a$. _____

2. If f is differentiable at $x = a$, then $\lim_{x \rightarrow a} f(x) = f(a)$. _____

3. If f has a jump discontinuity at $x = a$, then $f'(a)$ does not exist. _____

4. If $f'(a)$ does not exist, then f is not continuous at $x = a$. _____

5. Every polynomial is differentiable at every real number. _____

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8. One-Sided Derivatives with the Definition

Recall:

$$f'_-(a) = \lim_{h \rightarrow 0^-} \frac{f(a+h) - f(a)}{h}$$

$$f'_+(a) = \lim_{h \rightarrow 0^+} \frac{f(a+h) - f(a)}{h}$$

Use the definition to find both one-sided derivatives at the given point. Then state whether f is differentiable there and name the feature on the graph.

1.

$$f(x) = |x - 3|, \quad a = 3$$

2.

$$f(x) = x^{2/3}, \quad a = 0$$

3.

$$f(x) = x^{1/3}, \quad a = 0$$

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9. Is the Piecewise Function Differentiable?

For each function, check (i) continuity and (ii) equality of the one-sided derivatives at the break point.

1.

$$g(x) = \begin{cases} x^2, & x \leq 1 \\ 2x - 1, & x > 1 \end{cases}$$

2.

$$h(x) = \begin{cases} x^2 + 2x, & x < 0 \\ 3x, & x \geq 0 \end{cases}$$

3.

$$k(x) = \begin{cases} x^3, & x \leq 1 \\ 3x - 1, & x > 1 \end{cases}$$

4.

$$m(x) = \begin{cases} \sin x, & x < 0 \\ x - x^2, & x \geq 0 \end{cases}$$

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10. Find the Constants

Find the values of the constants that make each function differentiable everywhere. Show both the continuity and the derivative conditions.

1.

$$f(x) = \begin{cases} x^2, & x \leq 3 \\ mx + b, & x > 3 \end{cases}$$

2.

$$g(x) = \begin{cases} kx^2, & x \leq 2 \\ 2x + c, & x > 2 \end{cases}$$

3.

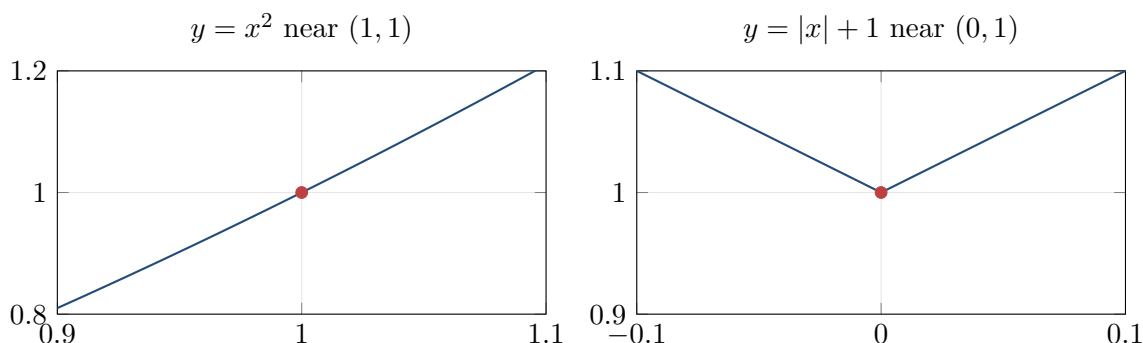
$$h(x) = \begin{cases} ax^3 + bx, & x \leq 1 \\ 4x^2 - 2, & x > 1 \end{cases}$$

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11. Local Linearity

A function that is differentiable at $x = a$ looks more and more like a straight line as you zoom in on the point $(a, f(a))$.



1. Both windows are zoomed in closely. Which graph looks like a straight line? Which still has a sharp point?

2. Explain how zooming in can help you decide whether a function is differentiable at a point.

12. Numerical Derivatives

Graphing calculators estimate $f'(a)$ with the *symmetric difference quotient*:

$$f'(a) \approx \frac{f(a+h) - f(a-h)}{2h}$$

1. For $f(x) = x^3$, use $h = 0.01$ to estimate $f'(2)$. Compare with the exact value $f'(2) = 12$.

2. For $f(x) = |x|$, compute the symmetric difference quotient at $a = 0$ with $h = 0.01$. Why is this result misleading?

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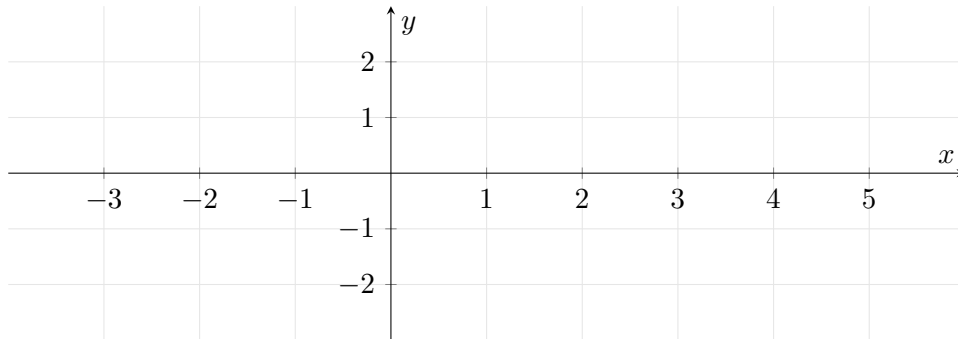
13. AP-Style ProblemLet f be the function defined by

$$f(x) = \begin{cases} \sqrt{x+3}, & -3 \leq x \leq 1 \\ 3-x, & 1 < x \leq 5 \end{cases}$$

1. Is
- f
- continuous at
- $x = 1$
- ? Justify using limits.

2. Find
- $\lim_{h \rightarrow 0^-} \frac{f(1+h) - f(1)}{h}$
- and
- $\lim_{h \rightarrow 0^+} \frac{f(1+h) - f(1)}{h}$
- . Is
- f
- differentiable at
- $x = 1$
- ?

3. Sketch the graph of
- f
- on the grid. Describe the behavior of the graph at
- $x = -3$
- .



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14. Derivatives Have the Intermediate Value Property

If f is differentiable on $[a, b]$, then f' takes on *every* value between $f'(a)$ and $f'(b)$, even if f' itself is not continuous.

1. The *step function*

$$U(x) = \begin{cases} 0, & x < 0 \\ 1, & x \geq 0 \end{cases}$$

jumps from 0 to 1. Can U be the derivative of any function on $(-\infty, \infty)$? Explain.

2. A differentiable function f has $f'(1) = -2$ and $f'(4) = 5$. Must there be a number c in $(1, 4)$ with $f'(c) = 0$? Explain what this means about the graph.

15. Explain Your Thinking

1. Explain why a function cannot be differentiable at a point where it has a jump. (Hint: what happens to the difference quotient on the side of the jump?)

2. The function $f(x) = x|x|$ is built from $|x|$, which has a corner at 0. Use the definition of the derivative to decide whether f is differentiable at $x = 0$.

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Mixed Review

16. Mixed Practice

Show your work clearly.

1. Find every x -value where $f(x) = |x^2 - 4x|$ is not differentiable.

2. Where is $g(x) = \frac{1}{x-2}$ not differentiable? Why?

3. Where is $h(x) = (x-1)^{1/3} + 2$ not differentiable? Name the feature.

4. Is $p(x) = \begin{cases} x^2 - 1, & x < 2 \\ 4x - 5, & x \geq 2 \end{cases}$ differentiable at $x = 2$?

17. Challenge Problem

Find a and b so that $f(x) = \begin{cases} a \cos x + b, & x \leq 0 \\ x^2 + 4x + 3, & x > 0 \end{cases}$ is differentiable at $x = 0$. If no such values exist, explain why.

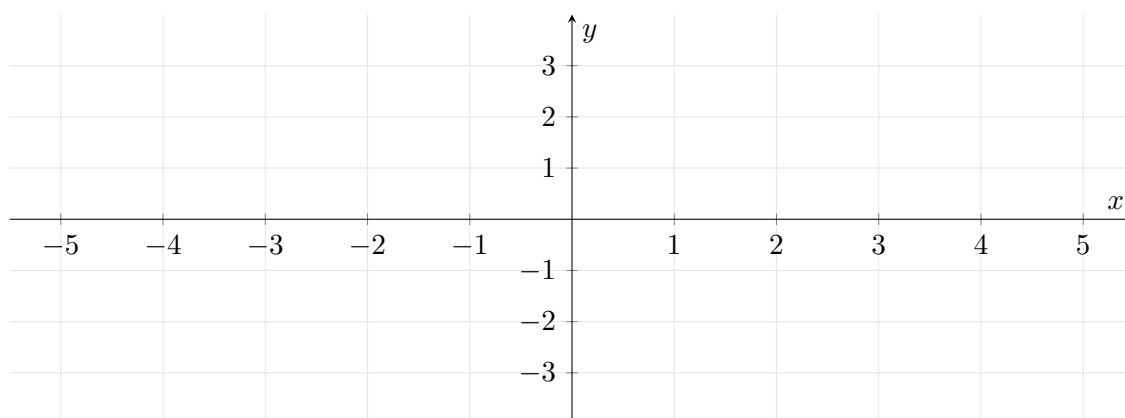
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18. Graphing Challenge

Sketch the graph of a function f on $[-5, 5]$ that satisfies all of the following:

- f is continuous on $[-5, 5]$
- f has a corner at $x = -3$
- f has a cusp at $x = 0$
- f has a vertical tangent at $x = 3$
- f is differentiable at every other x in $(-5, 5)$



Explain how your graph shows the difference between a cusp and a vertical tangent.

Reflection: What is one thing you understand better about differentiability after completing this worksheet?
