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Date: _____

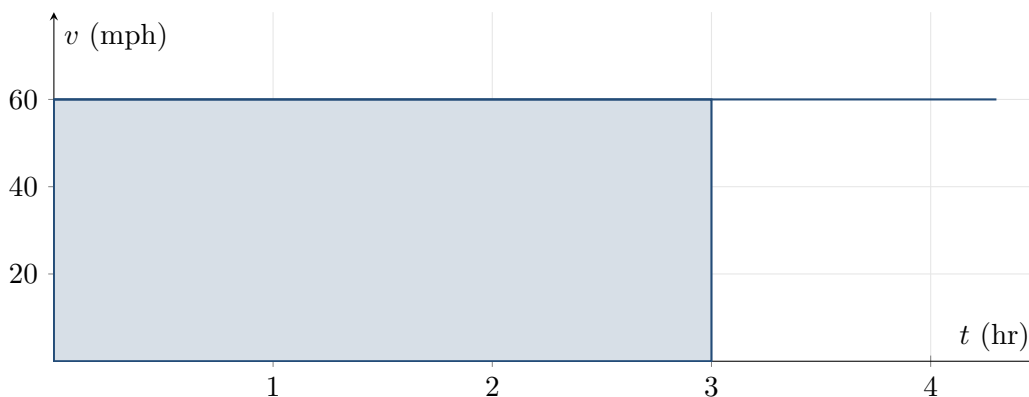
ESTIMATING WITH FINITE SUMS

AP Calculus AB Practice Worksheet

Learning Goal: Approximate accumulated quantities such as distance, volume, and total change using finite sums of rectangles (LRAM, RRAM, MRAM), decide whether an estimate is too high or too low, and interpret the results with correct units.

1. Distance as Area

A car travels at a constant velocity of 60 mph for 3 hours.



1. Find the distance traveled using distance = rate $\times$ time. _____

2. Find the area of the shaded rectangle. What are its units? _____

3. Explain why the area under a velocity graph gives distance traveled.

4. Suppose instead the car drives 60 mph for 1 hour and then 40 mph for 2 hours. Sketch the new velocity graph on the axes above (dashed) and find the total distance.

Name: _____

Date: _____

2. Distance with a Changing Velocity

A runner's velocity is recorded every 2 seconds.

t (sec)	0	2	4	6	8	10
$v(t)$ (ft/sec)	4	7	10	12	13	14

1. Estimate the distance traveled from $t = 0$ to $t = 10$ using the velocity at the **left** end of each interval.

Interval	$[0, 2]$	$[2, 4]$	$[4, 6]$	$[6, 8]$	$[8, 10]$
Velocity used					
Distance (ft)					

Left-endpoint estimate = _____ ft

2. Repeat using the velocity at the **right** end of each interval.

3. The runner's velocity is always increasing. Which estimate is too low and which is too high? Explain.

Name: _____

Date: _____

3. Rectangular Approximation Methods

To approximate the area under $y = f(x)$ on $[a, b]$, divide $[a, b]$ into n subintervals of equal width.

$$\Delta x = \frac{b - a}{n}$$

LRAM (left) Height of each rectangle = $f(\text{left endpoint})$

RRAM (right) Height of each rectangle = $f(\text{right endpoint})$

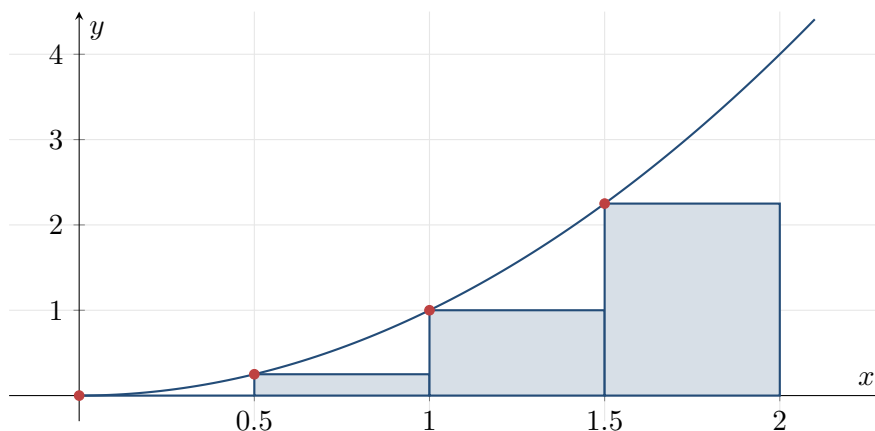
MRAM (midpoint) Height of each rectangle = $f(\text{midpoint})$

$$\text{Estimate} = \Delta x [f(x_1) + f(x_2) + \cdots + f(x_n)]$$

4. Left Rectangles (LRAM)

The graph shows $f(x) = x^2$ on $[0, 2]$ with $n = 4$ left-endpoint rectangles.

LRAM, $n = 4$



1. Find Δx and list the four left endpoints. _____

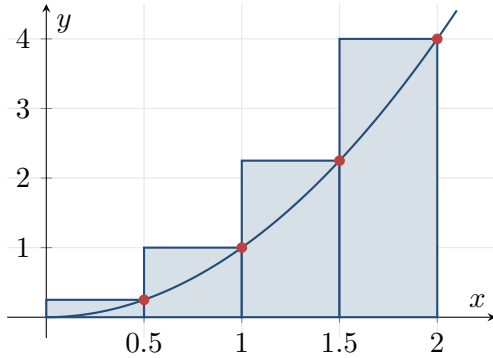
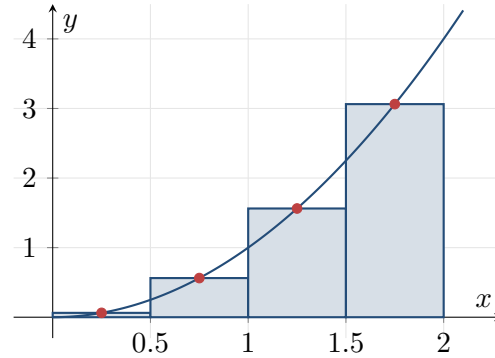
2. Compute LRAM_4 .

Name: _____

Date: _____

5. Right and Midpoint Rectangles

The same function $f(x) = x^2$ on $[0, 2]$ with $n = 4$.

 RRAM, $n = 4$

 MRAM, $n = 4$


1. Compute RRAM_4 .

2. Compute MRAM_4 .

3. The exact area is $\frac{8}{3} \approx 2.667$. Which of the three estimates (LRAM, RRAM, MRAM) is the closest? Which is an overestimate, and which are underestimates?

Name: _____

Date: _____

6. More Rectangles, Better Estimates

The table shows estimates for the area under $f(x) = x^2$ on $[0, 2]$. Fill in the $n = 4$ row with your answers from Sections 4 and 5.

n	LRAM_n	MRAM_n	RRAM_n
4			
8	2.1875	2.65625	3.1875
16	2.421875	2.6640625	2.921875
32	2.54296875	2.66601563	2.79296875
100	2.6268	2.6666	2.7068
1000	2.662668	2.666666	2.670668

- As n increases, what number do all three estimates appear to approach? _____
- Which method approaches the true area fastest? _____
- For each n , compute $\text{RRAM}_n - \text{LRAM}_n$. What pattern do you see? Write a formula for the difference in terms of n .

- Explain, in terms of rectangles, why the estimates improve as n increases.

Name: _____

Date: _____

7. Overestimate or Underestimate?

If f is ...	LRAM	RRAM
increasing on $[a, b]$	underestimate	overestimate
decreasing on $[a, b]$	overestimate	underestimate

For MRAM: if the graph is **concave up**, MRAM underestimates; if **concave down**, MRAM overestimates.

8. Classify the Estimate

For each function and interval, decide whether the estimate is an **overestimate** or an **underestimate** of the true area. Justify briefly.

Function	Interval	Method	Over/Under	Reason
$f(x) = \sqrt{x}$	$[0, 4]$	LRAM		
$f(x) = \frac{1}{x}$	$[1, 5]$	LRAM		
$f(x) = e^{-x}$	$[0, 3]$	RRAM		
$f(x) = x^3$	$[0, 2]$	RRAM		
$f(x) = x^2$	$[0, 2]$	MRAM		
$f(x) = \sqrt{x}$	$[1, 9]$	MRAM		

9. Upper and Lower Bounds

For $f(x) = \frac{1}{x}$ on $[1, 5]$ with $n = 4$:

1. Compute LRAM_4 and RRAM_4 as exact fractions.

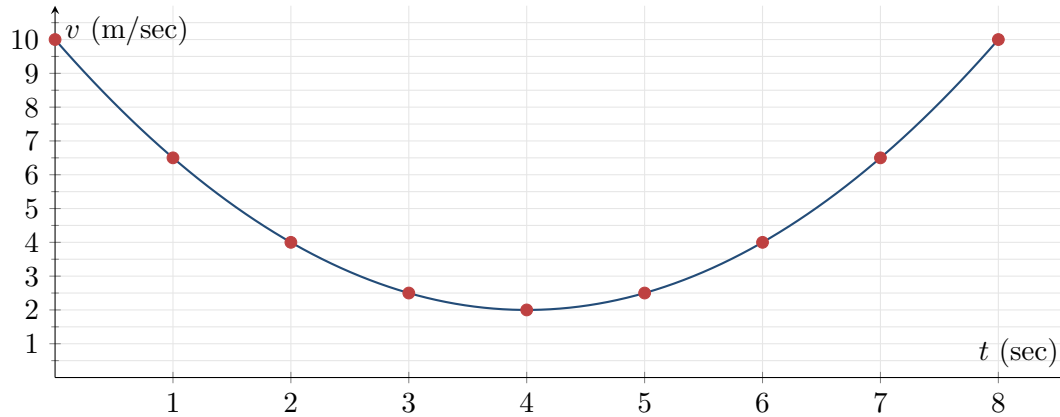
2. The true area lies between these two numbers. Write the inequality. _____

Name: _____

Date: _____

10. Distance from a Velocity Graph

The graph shows the velocity $v(t)$, in meters per second, of a particle moving along a line for $0 \leq t \leq 8$. The marked points lie exactly on half-units of the grid.



1. Use MRAM with 4 subintervals of equal width to estimate the distance traveled from $t = 0$ to $t = 8$.

2. Use LRAM with 4 subintervals of equal width to estimate the same distance.

3. The exact distance is $\frac{112}{3} \approx 37.33$ m. Which estimate is closer? Explain why, using the shape of the graph.

Name: _____

Date: _____

11. AP-Style Problem: Unequal Subintervals

Water flows into a tank at a rate of $R(t)$ gallons per hour, where t is measured in hours. Selected values of $R(t)$ are shown in the table.

t (hours)	0	2	5	7	10	12
$R(t)$ (gal/hr)	20	26	30	28	22	18

- The subintervals are not all the same width. List the width of each subinterval.

- Use a **right** Riemann sum with the five subintervals indicated by the table to estimate the total amount of water that flows into the tank from $t = 0$ to $t = 12$. Show the computation that leads to your answer.

- Use a **left** Riemann sum with the same subintervals to estimate the same quantity.

- Average your two estimates. Explain what your answer means in the context of the problem, with units.

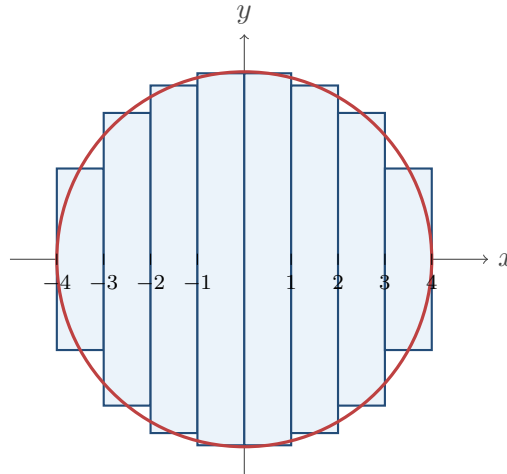
- Is it possible to tell whether the right Riemann sum is an overestimate or an underestimate? Explain.

Name: _____

Date: _____

12. Estimating the Volume of a Sphere

A sphere of radius 4 can be sliced into thin cylinders. Slice it into 8 cylinders of height 1 along the x -axis from $x = -4$ to $x = 4$. The cylinder centered at $x = m$ has radius $\sqrt{16 - m^2}$.



1. Complete the table. The volume of a cylinder is $V = \pi r^2 h$.

Midpoint m	-3.5	-2.5	-1.5	-0.5	0.5	1.5	2.5	3.5
$r^2 = 16 - m^2$								

2. Use MRAM to estimate the volume of the sphere. Leave your answer in terms of π , then give a decimal.

3. Compare with the exact volume $V = \frac{4}{3}\pi r^3$. Find the percent error.

Name: _____

Date: _____

13. Total Change from a Rate

An oil tanker is leaking. The rate of leakage is measured every hour.

Time (hr)	0	1	2	3	4	5
Leakage (gal/hr)	50	70	97	136	190	265

1. Give an upper estimate and a lower estimate of the total amount of oil that leaked during the first 5 hours.

2. The leak continues and the rates for hours 5 through 8 are 265, 369, 516, and 720 gal/hr. Find upper and lower estimates for the oil leaked in the first 8 hours.

14. Explain Your Thinking

1. Under what condition on $f(a)$ and $f(b)$ will LRAM_n and RRAM_n give the *same* value for every n ? (Hint: compare the two sums term by term.)

2. A student says, “MRAM is always the best of the three methods.” Is this true? Explain.

Name: _____

Date: _____

ESTIMATING WITH FINITE SUMS

Mixed Review

15. Mixed Practice

Show your work clearly.

1. Estimate the area under $f(x) = 4 - x^2$ on $[0, 2]$ using RRAM_4 .

2. Estimate the area under $f(x) = x^3$ on $[1, 3]$ using MRAM_2 .

3. Estimate the area under $f(x) = \sin x$ on $[0, \pi]$ using LRAM_4 . Give an exact answer and a decimal.

4. A car's speedometer reads 30, 42, 50, and 46 mph at $t = 0, 0.25, 0.5$, and 0.75 hours. Estimate the distance traveled in the first hour using left endpoints.

16. Challenge Problem

For $f(x) = x^2$ on $[0, 2]$, show that $\text{RRAM}_n - \text{LRAM}_n = \frac{8}{n}$. How large must n be so that the two estimates differ by less than 0.01?

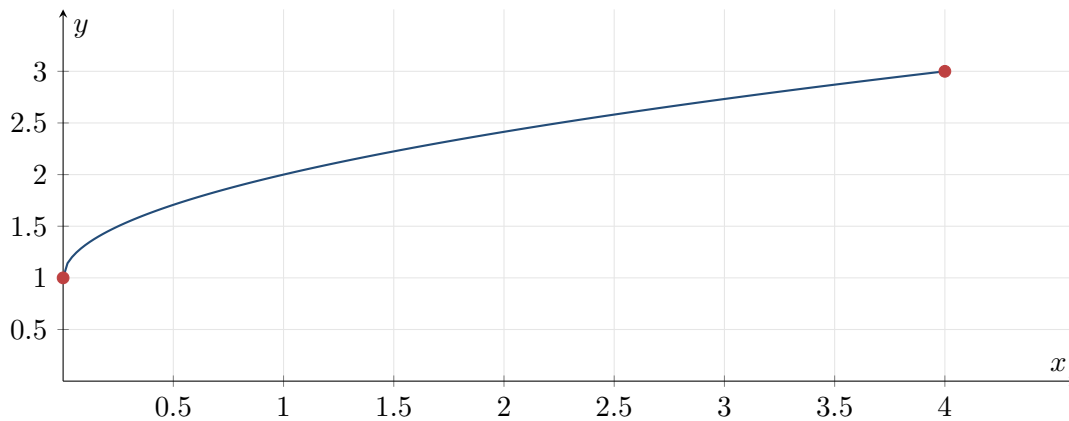
Name: _____

Date: _____

17. Graphing Challenge

The graph of $g(x) = 1 + \sqrt{x}$ on $[0, 4]$ is shown.

- Draw the 4 rectangles for MRAM_4 on the grid.
- Mark the midpoint height on each rectangle.
- Use your rectangles to estimate the area under g on $[0, 4]$.



1. $\text{MRAM}_4 \approx$ _____

2. Is your estimate too high or too low? Use the shape of the graph to explain.

Reflection: What is one thing you understand better about estimating with finite sums after completing this worksheet?
