

Name: _____

Date: _____

DERIVATIVES OF EXPONENTIAL & LOGARITHMIC FUNCTIONS

AP Calculus AB Practice Worksheet

Learning Goal: Differentiate exponential and logarithmic functions of any base, combine these rules with the chain, product, and quotient rules, use logarithmic differentiation, and interpret the results as rates of change.

1. Why Is e Special?

Complete the table for

$$\frac{e^h - 1}{h}.$$

Round each value to four decimal places.

h	-0.1	-0.01	-0.001	0.001	0.01	0.1
$\frac{e^h - 1}{h}$						

$$\lim_{h \rightarrow 0} \frac{e^h - 1}{h} = \underline{\hspace{2cm}}$$

2. The Derivative of e^x

Use the definition of the derivative to complete the steps.

1. Fill in the blanks:

$$\frac{d}{dx}(e^x) = \lim_{h \rightarrow 0} \frac{e^{x+h} - e^x}{h} = \lim_{h \rightarrow 0} \frac{e^x (\underline{\hspace{1cm}})}{h} = e^x \cdot \lim_{h \rightarrow 0} \frac{\underline{\hspace{1cm}}}{h} = \underline{\hspace{2cm}}$$

2. What does your result tell you about the slope of $y = e^x$ at any point compared with the y -coordinate of that point?

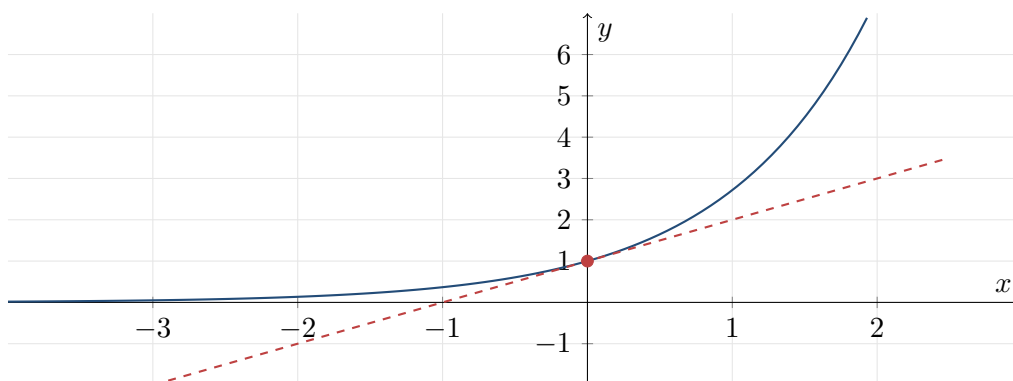
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Name: _____

Date: _____

3. Read the Slope from the Graph

The graph of $y = e^x$ is shown below, together with its tangent line (dashed) at the point $(0, 1)$.



Answer the following:

1. Use the tangent line to find the slope of $y = e^x$ at $x = 0$.
2. Write the equation of the dashed tangent line.
3. Find the exact slope of $y = e^x$ at $x = 1$ and at $x = \ln 5$.

4. Is $y = e^x$ ever decreasing? Use its derivative to explain.

4. Derivative Rules

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(e^u) = e^u \frac{du}{dx}$$

$$\frac{d}{dx}(a^x) = a^x \ln a$$

$$\frac{d}{dx}(a^u) = a^u \ln a \frac{du}{dx}$$

$$\frac{d}{dx}(\ln x) = \frac{1}{x}, \quad x > 0$$

$$\frac{d}{dx}(\ln u) = \frac{1}{u} \frac{du}{dx}$$

$$\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$$

$$\frac{d}{dx}(\log_a u) = \frac{1}{u \ln a} \frac{du}{dx}$$

Name: _____

Date: _____

5. Derivatives of Natural Exponential FunctionsFind $\frac{dy}{dx}$.

1.

$$y = e^{5x}$$

2.

$$y = e^{x^2-3x}$$

3.

$$y = x^3 e^x$$

4.

$$y = \frac{e^x}{x+1}$$

5.

$$y = e^{\sin x}$$

Name: _____

Date: _____

6. Derivatives of a^x

Find the derivative. Leave your answers in exact form.

1. $y = 3^x$

2. $y = 5^{2x+1}$

3. $y = 2^{x^2}$

4. $y = x \cdot 10^x$

7. Rewrite with Base e Since $a = e^{\ln a}$, every exponential can be written as $a^x = e^{x \ln a}$. Use this fact to show that

$$\frac{d}{dx}(a^x) = a^x \ln a.$$

Name: _____

Date: _____

8. Derivatives of Natural Logarithms

Find $\frac{dy}{dx}$. State any restrictions on x .

1.

$$y = \ln(4x)$$

2.

$$y = \ln(x^2 + 1)$$

3.

$$y = x^2 \ln x$$

4.

$$y = \ln(\cos x)$$

5.

$$y = (\ln x)^3$$

Name: _____

Date: _____

9. Use Log Properties First

Simplify using properties of logarithms, then differentiate.

$$\ln(ab) = \ln a + \ln b \quad \ln\left(\frac{a}{b}\right) = \ln a - \ln b \quad \ln(a^n) = n \ln a$$

1. $y = \ln\left(\frac{x^2}{x+5}\right)$

2. $y = \ln(x^3\sqrt{2x-1})$

10. Derivatives of $\log_a x$

Find the derivative.

1. $y = \log_2 x$

2. $y = \log_{10}(3x^2 + 1)$

3. $y = \log_5(\sin x)$

Name: _____

Date: _____

11. Logarithmic Differentiation

Use logarithmic differentiation when the variable appears in both the base and the exponent, or when a function is a complicated product or quotient.

1. Take $\ln$ of both sides.
2. Use log properties to simplify.
3. Differentiate implicitly.
4. Solve for $\frac{dy}{dx}$ and substitute for y .

1.

$$y = x^x, \quad x > 0$$

2.

$$y = x^{\sin x}, \quad x > 0$$

3.

$$y = \frac{(x^2 + 1)^3 \sqrt{x - 1}}{(x + 4)^2}$$

Name: _____

Date: _____

12. Power Rule for Arbitrary Real PowersFor any real number n and $x > 0$:

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Find the derivative.

1. $y = x^{\sqrt{2}}$

2. $y = x^\pi + \pi^x$

13. Equations of Tangent Lines

Write an equation of the tangent line at the given point.

1. $y = e^{2x}, \quad x = 0$

2. $y = \ln x, \quad x = e$

14. Explain Your ThinkingUse derivatives to explain why the graph of $y = \ln x$ is always increasing, but becomes less steep as x gets larger.

Name: _____

Date: _____

15. Horizontal Tangent Lines

Find every x -value where the graph has a horizontal tangent line.

1. $f(x) = xe^{-x}$

2. $g(x) = x \ln x$

16. Rates of Change in Context

Include units in each answer.

1. The population of a town is modeled by

$$P(t) = 5000e^{0.03t},$$

where t is measured in years. Find $P'(10)$ and explain its meaning.

2. A radioactive sample decays according to

$$A(t) = 80 \left(\frac{1}{2}\right)^{t/5},$$

where A is in grams and t is in days. Find the rate of decay at $t = 10$ days.

Name: _____

Date: _____

EXPONENTIAL & LOGARITHMIC DERIVATIVES

Mixed Review

17. Mixed Practice

Show your work clearly.

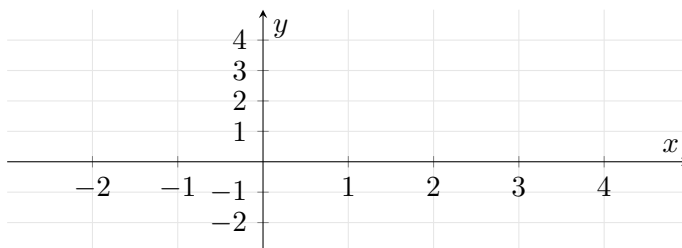
1. Find
- $\frac{dy}{dx}$
- for
- $y = e^{-x} \cos x$
- .

2. Find
- $\frac{dy}{dx}$
- for
- $y = \ln(e^x + x^2)$
- .

18. Challenge ProblemFind all values of k for which $y = e^{kx}$ satisfies $y'' - 5y' + 6y = 0$.**19. Graphing Challenge**

On the grid:

- Sketch $y = e^x$ and $y = \ln x$
- Draw the line $y = x$
- Draw the tangent to $y = e^x$ at $(0, 1)$
- Draw the tangent to $y = \ln x$ at $(1, 0)$



What do you notice?

Reflection: What is one thing you understand better about derivatives of exponential and logarithmic functions after completing this worksheet?