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EXTREME VALUES OF FUNCTIONS

AP Calculus AB Practice Worksheet

Learning Goal: Identify absolute and local extreme values from graphs, tables, and formulas, apply the Extreme Value Theorem, find critical points, and use the Candidates Test to locate absolute extrema on a closed interval.

1. Absolute and Local Extreme Values

Absolute maximum $f(c)$ is an absolute max on D if $f(c) \geq f(x)$ for all x in D .

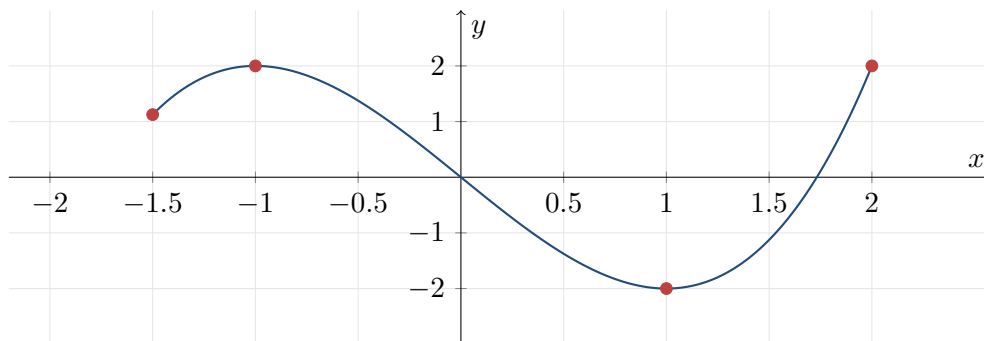
Absolute minimum $f(c)$ is an absolute min on D if $f(c) \leq f(x)$ for all x in D .

Local maximum $f(c) \geq f(x)$ for all x in some open interval containing c .

Local minimum $f(c) \leq f(x)$ for all x in some open interval containing c .

2. Read Extreme Values from a Graph

The graph of $f(x) = x^3 - 3x$ on the closed interval $[-1.5, 2]$ is shown.



- The absolute maximum value is _____, and it occurs at $x =$ _____.
- The absolute minimum value is _____, and it occurs at $x =$ _____.
- List any local extreme values in the open interval $(-1.5, 2)$.

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3. The Extreme Value Theorem

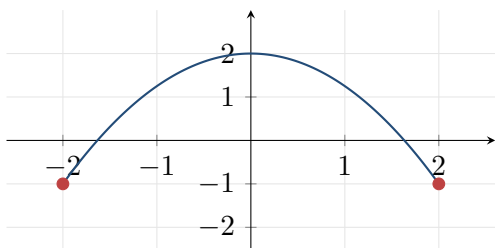
Extreme Value Theorem (EVT): If f is **continuous** on a **closed** interval $[a, b]$, then f has both an absolute maximum value and an absolute minimum value on $[a, b]$.

Both hypotheses matter. If either one fails, the extreme values *might* not exist.

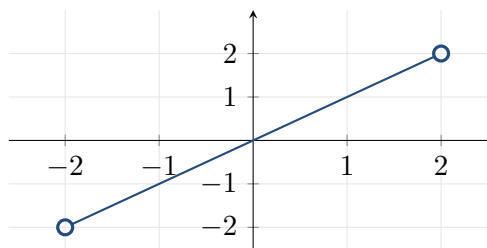
4. Does the Theorem Apply?

For each graph, decide whether the hypotheses of the EVT are met, and whether f actually has an absolute maximum and an absolute minimum.

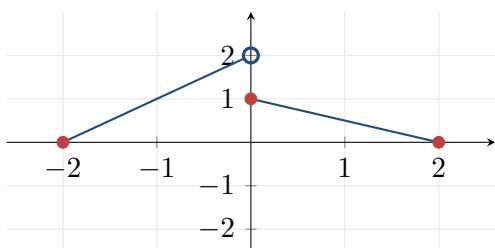
A: $[-2, 2]$



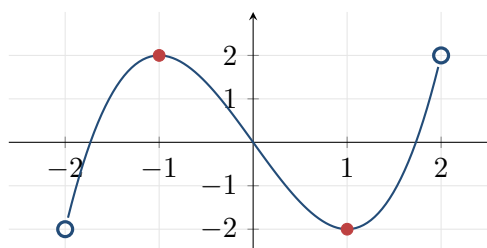
B: $(-2, 2)$



C: $[-2, 2]$



D: $(-2, 2)$



Graph	EVT applies? (Y/N)	Absolute max?	Absolute min?
A			
B			
C			
D			

Graph D does not satisfy the EVT, yet it has both extreme values. Explain why this does not contradict the theorem.

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5. Critical Points

A **critical point** of f is a point c in the domain of f where

$$f'(c) = 0 \quad \text{or} \quad f'(c) \text{ does not exist.}$$

Local Extreme Value Theorem: If f has a local max or min at an interior point c and $f'(c)$ exists, then $f'(c) = 0$.

6. Find the Critical Points

Find all critical points of each function. State whether $f' = 0$ or f' is undefined at each.

1. $f(x) = x^3 - 12x + 4$

2. $g(x) = x^4 - 8x^2$

3. $h(x) = x^{2/3}(x - 5)$

4. $k(x) = \frac{x}{x^2 + 1}$

5. $m(x) = \sin x + \cos x, \quad 0 \leq x \leq 2\pi$

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7. The Candidates Test

To find the absolute extrema of a continuous function f on $[a, b]$:

1. Find all critical points of f in (a, b) .
2. Evaluate f at each critical point and at the endpoints a and b .
3. The largest value is the absolute max; the smallest is the absolute min.

8. Use a Candidates Table

Let

$$f(x) = 2x^3 - 3x^2 - 12x + 5 \quad \text{on} \quad [-2, 4].$$

1. Find $f'(x)$ and the critical points in $(-2, 4)$.

2. Complete the candidates table.

Candidate x	-2			4
Type	endpoint	critical pt.	critical pt.	endpoint
$f(x)$				

3. State the absolute maximum and absolute minimum values and where each occurs.

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9. Absolute Extrema on a Closed Interval

Find the absolute maximum and minimum values of each function on the given interval. Show your candidates.

1.

$$f(x) = x^3 - 3x^2 + 1, \quad [-1, 3]$$

2.

$$g(x) = x^{2/3}, \quad [-1, 8]$$

3.

$$h(x) = x + 2 \cos x, \quad [0, \pi]$$

4.

$$k(x) = \ln(x^2 + 1), \quad [-1, 2]$$

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10. More Absolute Extrema

Find the absolute extreme values. Give exact answers.

1.

$$f(x) = xe^{-x}, \quad [0, 3]$$

2.

$$g(x) = \frac{x}{x^2 + 4}, \quad [0, 3]$$

3.

$$h(x) = \sqrt{9 - x^2}, \quad [-2, 3]$$

4.

$$p(x) = |x - 2| + 1, \quad [0, 5]$$

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11. When Extreme Values May Not Exist

Decide whether each function has an absolute maximum and an absolute minimum on the given interval. If one exists, give its value. If not, explain why.

1. $f(x) = x^2, \quad (-1, 2)$

2. $g(x) = \frac{1}{x}, \quad (0, 1]$

3. $h(x) = e^x, \quad (-\infty, \infty)$

4. $k(x) = \begin{cases} x + 1, & -1 \leq x < 1 \\ 0, & x = 1 \end{cases} \quad \text{on } [-1, 1]$

12. Explain Your Thinking

Is every critical point the location of a local extreme value? Use $f(x) = x^3$ to support your answer.

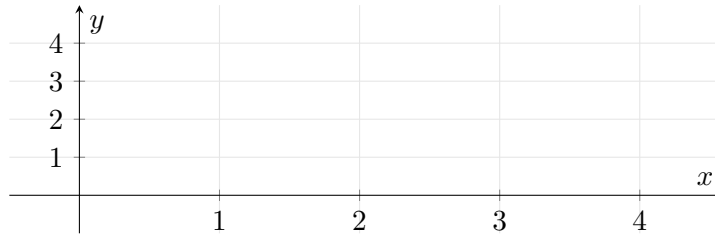
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13. Sketch a Function

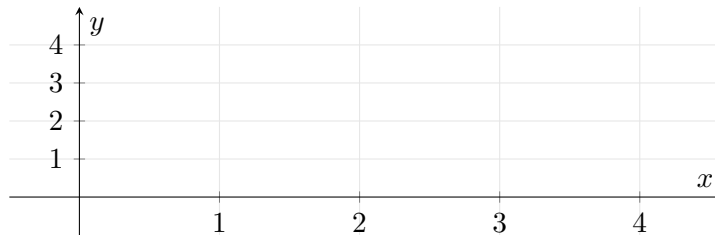
Sketch a graph that satisfies the given conditions, or explain why it is impossible.

- f is continuous on $[0, 4]$, has an absolute maximum at $x = 4$, and has an absolute minimum at $x = 2$ where $f'(2) = 0$.

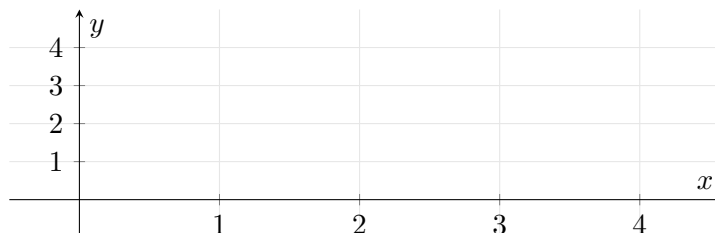


- f is continuous on $[0, 4]$, but has *no* absolute maximum.

- f is defined on $[0, 4]$, has an absolute minimum, and has *no* absolute maximum.



- f is continuous on $[0, 4]$ with a critical point at $x = 1$ where $f'(1)$ does not exist, and the absolute maximum occurs at $x = 1$.



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14. Extreme Values in Context

1. A particle moves along the x -axis. Its position at time t seconds is

$$s(t) = t^3 - 6t^2 + 9t + 2, \quad 0 \leq t \leq 5.$$

Find the particle's greatest and least positions on $[0, 5]$ and the time(s) at which each occurs.

2. A store sells headphones for p dollars each. Its weekly revenue is

$$R(p) = p(100 - 2p), \quad 0 \leq p \leq 50.$$

What price gives the greatest revenue? What is that revenue?

3. The temperature (in $^{\circ}\text{F}$) t hours after midnight is modeled by

$$T(t) = 60 - 12 \cos\left(\frac{\pi t}{12}\right), \quad 0 \leq t \leq 24.$$

Find the warmest and coolest temperatures and when they occur.

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15. AP-Style Problem: Using a Table

The function f is differentiable on $[0, 6]$. The only values of x where $f'(x) = 0$ are $x = 1$ and $x = 4$. Selected values of f are shown.

x	0	1	4	6
$f(x)$	2	7	-3	4

- Find the absolute maximum and absolute minimum values of f on $[0, 6]$. Justify your answer.

- Explain why the Extreme Value Theorem guarantees that these values exist.

- Must there be a value c in $(1, 4)$ with $f(c) = 0$? Justify, naming the theorem you use.

16. Reading Extrema from f'

Suppose $f'(x) = (x + 2)(x - 1)^2$ and f is defined for all real numbers.

- List the critical points of f . _____
- At which critical point does f change from decreasing to increasing? What kind of extreme value occurs there?

- Why is there no local extreme value at $x = 1$?

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EXTREME VALUES OF FUNCTIONS

Mixed Review

17. Mixed Practice

Show your work clearly.

1. Find the critical points of
- $f(x) = 3x^4 - 4x^3$
- .

2. Find the absolute extrema of
- $g(x) = x - 2\sqrt{x}$
- on
- $[0, 9]$
- .

3. Find the absolute extrema of
- $h(x) = \sin(2x)$
- on
- $[0, \frac{\pi}{2}]$
- .

4. Find the absolute extrema of
- $k(x) = x^2 - 4|x|$
- on
- $[-3, 1]$
- .

18. Challenge Problem

Find the values of a and b so that $f(x) = x^3 + ax^2 + b$ has a local extreme value at the point $(2, 3)$. Is it a local maximum or a local minimum?

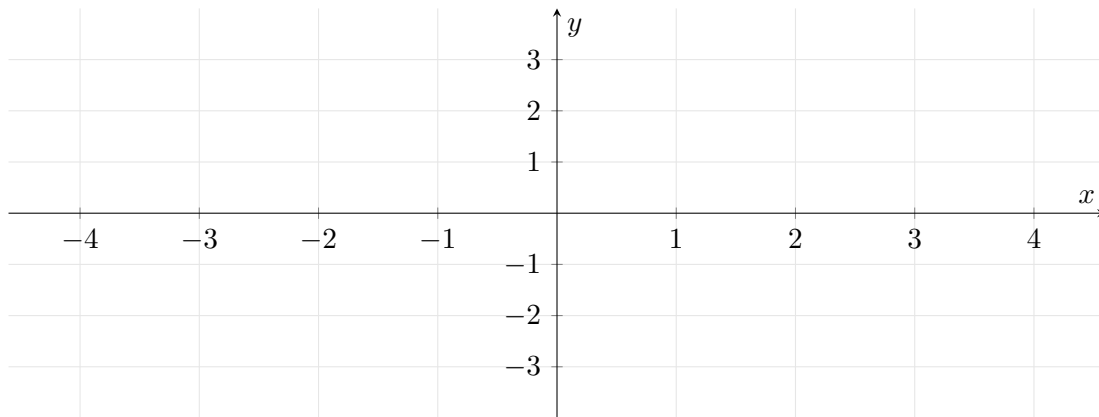
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19. Graphing Challenge

Sketch the graph of a continuous function f on $[-4, 4]$ that satisfies all of the following:

- $f(-4) = 1$ and $f(4) = -1$
- The absolute maximum value is 3, at $x = -2$, where $f'(-2) = 0$
- The absolute minimum value is -3 , at $x = 1$, where $f'(1)$ does not exist
- $f'(3) = 0$, but f has no local extreme value at $x = 3$



List all of the critical points of your function and classify each one.

Reflection: What is one thing you understand better about extreme values of functions after completing this worksheet?
