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FUNDAMENTAL THEOREM OF CALCULUS

AP Calculus AB Practice Worksheet

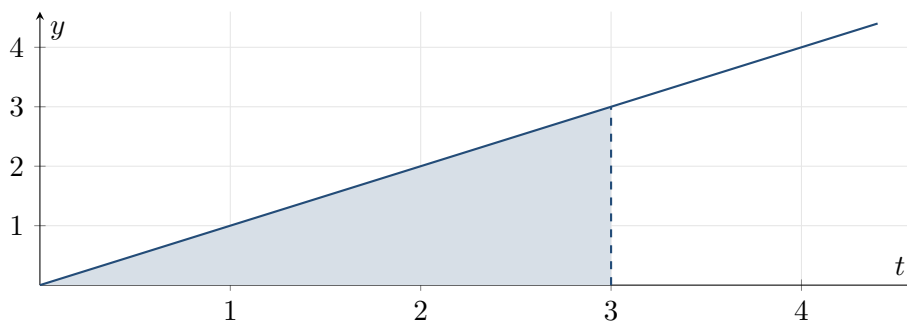
Learning Goal: Understand accumulation functions, differentiate integrals with variable limits (FTC Part 1), evaluate definite integrals using antiderivatives (FTC Part 2), and analyze functions defined by integrals using graphs and tables.

1. Discover the Connection

Let $f(t) = t$ and define the *accumulation function*

$$A(x) = \int_0^x t \, dt,$$

the area under $y = t$ from $t = 0$ to $t = x$.



1. Use triangle areas to complete the table.

x	0	1	2	3	4
$A(x)$					

2. Write a formula for $A(x)$. Then find $A'(x)$. How does $A'(x)$ compare with $f(x)$?

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2. The Fundamental Theorem of Calculus, Part 1

If f is continuous on $[a, b]$, then $F(x) = \int_a^x f(t) dt$ is differentiable on (a, b) and

$$\frac{d}{dx} \int_a^x f(t) dt = f(x)$$

Every continuous function has an antiderivative, and differentiation “undoes” integration.

3. Apply FTC Part 1

Find each derivative.

1. $\frac{d}{dx} \int_1^x (t^3 + 1) dt$

2. $\frac{d}{dx} \int_0^x \cos(t^2) dt$

3. $\frac{d}{dx} \int_2^x e^{-t^2} dt$

4. $\frac{d}{dx} \int_x^5 \sqrt{1+t^2} dt$ (Hint: switch the limits first.)

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4. FTC Part 1 with the Chain Rule

$$\frac{d}{dx} \int_a^{u(x)} f(t) dt = f(u(x)) \cdot u'(x)$$

$$\frac{d}{dx} \int_{v(x)}^{u(x)} f(t) dt = f(u(x)) \cdot u'(x) - f(v(x)) \cdot v'(x)$$

Find each derivative.

1.

$$\frac{d}{dx} \int_0^{x^2} \sin t dt$$

2.

$$\frac{d}{dx} \int_1^{3x} \ln t dt$$

3.

$$\frac{d}{dx} \int_{\sin x}^1 \frac{1}{1+t^2} dt$$

4.

$$\frac{d}{dx} \int_x^{x^2} t^3 dt$$

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5. The Fundamental Theorem of Calculus, Part 2

If f is continuous on $[a, b]$ and F is *any* antiderivative of f , then

$$\int_a^b f(x) \, dx = F(b) - F(a) = \left[F(x) \right]_a^b$$

6. Evaluate Definite Integrals

Evaluate each integral exactly.

1.

$$\int_1^3 (x^2 - 2x) \, dx$$

2.

$$\int_0^\pi \sin x \, dx$$

3.

$$\int_1^4 \sqrt{x} \, dx$$

4.

$$\int_0^1 e^x \, dx$$

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7. More Definite Integrals

Evaluate each integral exactly. Rewrite the integrand first when it helps.

1.

$$\int_1^e \frac{1}{x} dx$$

2.

$$\int_0^{\pi/4} \sec^2 x dx$$

3.

$$\int_0^1 \frac{1}{1+x^2} dx$$

4.

$$\int_1^2 \frac{x^3 + 2}{x^2} dx$$

5.

$$\int_{-1}^2 |x| dx \quad (\text{Hint: split the integral at } x = 0.)$$

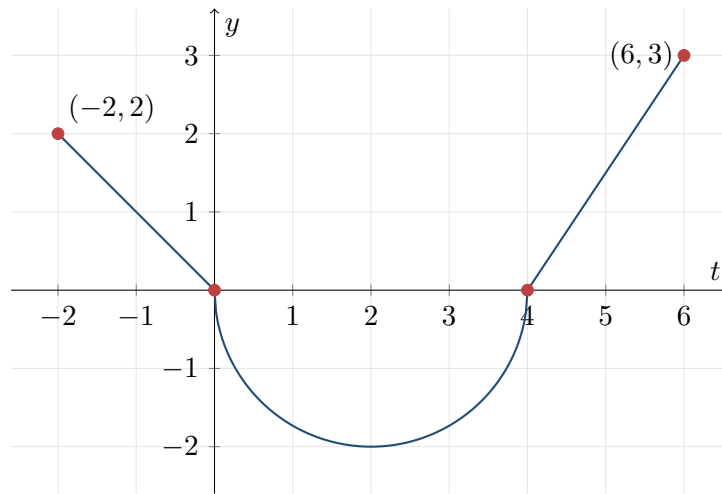
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8. Accumulation Functions from a Graph

The graph of a continuous function f on $[-2, 6]$ consists of two line segments and a semicircle, as shown. Let

$$g(x) = \int_0^x f(t) dt.$$



1. Find each value. Use geometry.

x	-2	0	2	4	6
$g(x)$					

2. Find $g'(-1)$, $g'(2)$, and $g'(5)$.

3. On what intervals is g increasing? Justify your answer.

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9. Analyze the Accumulation Function

Continue to use $g(x) = \int_0^x f(t) dt$ and the graph of f from Section 8.

1. Find the x -coordinate of each relative extremum of g on $(-2, 6)$. Classify each one and justify your answer.

2. Find the absolute maximum and absolute minimum values of g on $[-2, 6]$. Justify your answer.

3. On what intervals is the graph of g concave up? Concave down? Justify.

4. Find the x -coordinate of each point of inflection of the graph of g .

5. Let $h(x) = \int_2^x f(t) dt$. How are the graphs of g and h related? Find $h(4)$.

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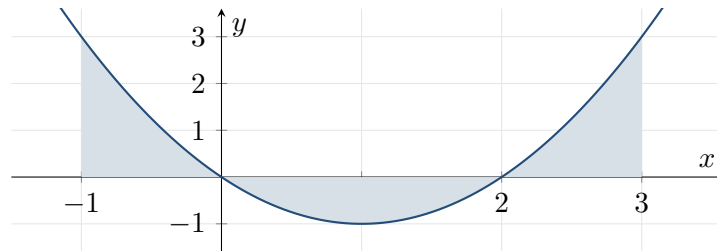
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10. Net Area and Total Area

$$\text{Net area} = \int_a^b f(x) dx \qquad \text{Total area} = \int_a^b |f(x)| dx$$

To find total area, split the interval at the zeros of f and add the absolute values.

1. Let $f(x) = x^2 - 2x$ on $[-1, 3]$.



Find the net area and the total area between the graph and the x -axis.

2. Find the net area and the total area between $y = \sin x$ and the x -axis on $[0, 2\pi]$.

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11. Net Change in Context**Net Change Theorem:** The integral of a rate of change is the total change:

$$\int_a^b F'(x) dx = F(b) - F(a)$$

1. A particle moves along a line with velocity

$$v(t) = 3t^2 - 12t + 9 \text{ m/sec}, \quad 0 \leq t \leq 5.$$

- (a) Find the displacement of the particle from $t = 0$ to $t = 5$.

- (b) Find the total distance traveled from $t = 0$ to $t = 5$.

- (c) The particle is at position $x = 5$ when $t = 2$. Find its position at $t = 5$.

2. Water leaks from a tank at a rate of $r(t) = 4e^{-0.2t}$ gallons per hour. How many gallons leak out during the first 5 hours? Give an exact answer and a decimal.

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12. AP-Style Problem: Using a Table

The function f is continuous on $[1, 5]$. Selected values of f are shown. Let

$$g(x) = \int_1^x f(t) dt.$$

x	1	2	3	4	5
$f(x)$	2	5	3	-1	4

- Find $g(1)$ and $g'(3)$. _____
- Use a right Riemann sum with four subintervals of equal width to approximate $g(5)$.

- Is g increasing or decreasing at $x = 4$? Justify your answer.

- Must there be a value c in $(3, 4)$ where g has a critical point? Explain.

- Let $h(x) = \int_1^{x^2} f(t) dt$. Find $h'(2)$.

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13. Work Backward

1. Find a function f and a number a such that

$$\int_a^x f(t) dt = x^2 - 2x - 3 \quad \text{for all } x.$$

(Hint: differentiate both sides, then substitute $x = a$.)

2. Find a function $y = F(x)$ with $F'(x) = \cos(x^2)$ and $F(1) = 4$. Write your answer using an integral.

14. Explain Your Thinking

1. Explain why $\frac{d}{dx} \int_a^x f(t) dt$ does not depend on the lower limit a .

2. A student writes $\int_{-1}^1 \frac{1}{x^2} dx = \left[-\frac{1}{x} \right]_{-1}^1 = -2$. Explain why this is wrong. (Hint: check the hypotheses of FTC Part 2.)

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Mixed Review

15. Mixed Practice

Show your work clearly.

1. Find $\frac{d}{dx} \int_3^{\sqrt{x}} \frac{t}{t^4 + 1} dt$.

2. Evaluate $\int_0^{\pi/3} (2 \cos x - \sec x \tan x) dx$.

3. Evaluate $\int_1^8 x^{-1/3} dx$.

4. If $\int_0^4 f(x) dx = 10$ and F is an antiderivative of f with $F(0) = 3$, find $F(4)$.

16. Challenge Problem

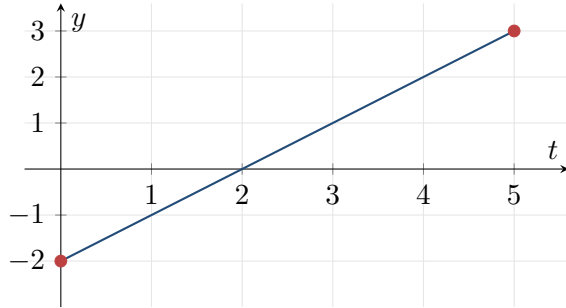
Let $G(x) = \int_0^x \frac{t^2 - 4}{1 + \cos^2 t} dt$. Find the x -coordinates of all relative extrema of G and classify each one. Justify your answer.

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17. Graphing Challenge

The graph of $f(t) = t - 2$ is shown. Let $g(x) = \int_0^x f(t) dt$ for $0 \leq x \leq 5$.

Graph of f Sketch g here

1. Use geometry to find $g(1)$, $g(2)$, $g(3)$, $g(4)$, and $g(5)$, and plot them on the right-hand grid.

2. Where does g have its minimum? How can you see this from the graph of f ?

3. Find a formula for $g(x)$ using FTC Part 2, and check it against your points.

Reflection: What is one thing you understand better about the Fundamental Theorem of Calculus after completing this worksheet?
