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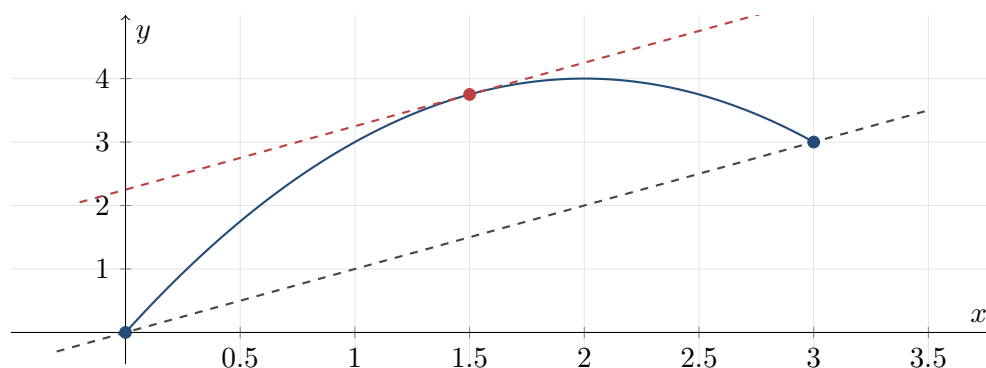
MEAN VALUE THEOREM

AP Calculus AB Practice Worksheet

Learning Goal: State, verify, and apply the Mean Value Theorem and Rolle's Theorem, find the values of c that the theorem guarantees, interpret the theorem in context, and use its corollaries to analyze increasing and decreasing functions.

1. Discover the Idea

The graph of $f(x) = -x^2 + 4x$ on $[0, 3]$ is shown with the secant line through its endpoints (gray, dashed) and a parallel tangent line (red, dashed).



- Find the slope of the secant line through $(0, f(0))$ and $(3, f(3))$. _____
- Find the value of c in $(0, 3)$ where $f'(c)$ equals the slope of the secant line.

- In your own words, describe the relationship between the two dashed lines.

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2. The Mean Value Theorem and Rolle's Theorem

Mean Value Theorem (MVT): If f is **continuous** on $[a, b]$ and **differentiable** on (a, b) , then there is at least one number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Rolle's Theorem (a special case): if, in addition, $f(a) = f(b)$, then there is at least one c in (a, b) with $f'(c) = 0$.

Geometric meaning	Some tangent line is parallel to the secant line.
Rate meaning	At some instant, the instantaneous rate equals the average rate.

3. Find the Value(s) of c

Verify that each function satisfies the hypotheses of the MVT on the interval. Then find every value of c guaranteed by the theorem.

1.

$$f(x) = x^2 + 2x, \quad [0, 2]$$

2.

$$f(x) = x^3, \quad [-1, 2]$$

3.

$$f(x) = \sqrt{x}, \quad [0, 4]$$

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4. More Values of c

Find every value of c guaranteed by the MVT or Rolle's Theorem. Give exact answers, then decimals where helpful.

1.

$$f(x) = \frac{1}{x}, \quad [1, 3]$$

2.

$$f(x) = \ln x, \quad [1, e]$$

3.

$$f(x) = \sin x, \quad \left[0, \frac{\pi}{2}\right]$$

4. (Rolle's Theorem)

$$f(x) = x^2 - 4x + 3, \quad [1, 3]$$

5. (Rolle's Theorem)

$$f(x) = x^3 - 3x, \quad \left[-\sqrt{3}, \sqrt{3}\right]$$

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5. Check the Hypotheses

Decide whether the MVT applies to each function on the given interval. Complete the table, then explain each “No.”

Function	Interval	Continuous on $[a, b]$?	Differentiable on (a, b) ?	MVT applies?
$f(x) = x $	$[-1, 2]$			
$f(x) = x^{2/3}$	$[-1, 8]$			
$f(x) = \frac{1}{x-2}$	$[0, 4]$			
$f(x) = x^{1/3}$	$[0, 8]$			
$f(x) = \begin{cases} x^2, & x < 1 \\ 2x - 1, & x \geq 1 \end{cases}$	$[0, 2]$			

1. For $f(x) = |x|$ on $[-1, 2]$, show that *no* value of c satisfies the conclusion of the MVT. Why doesn't this contradict the theorem?

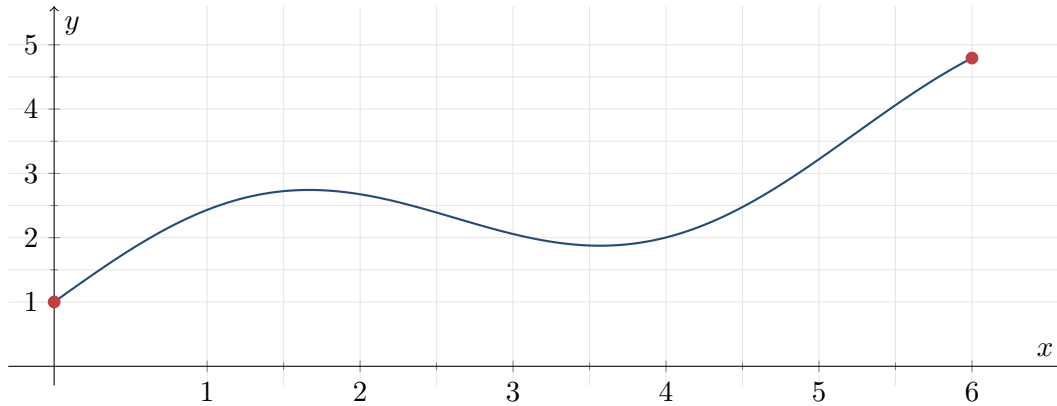
2. $f(x) = x^{1/3}$ has a vertical tangent at $x = 0$. Explain why the MVT still applies on $[0, 8]$, and find c .

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6. The MVT on a Graph

The graph of a differentiable function f on $[0, 6]$ is shown.



1. Draw the secant line through the endpoints. Estimate its slope. _____
2. Draw every tangent line that is parallel to the secant line. How many values of c satisfy the MVT on $[0, 6]$? Estimate each one.

3. Does the MVT tell you how many values of c there are? Explain.

4. Is there a value of c in $(0, 6)$ where $f'(c) = 0$? How do you know from the graph?

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7. The MVT in Motion

1. A car travels 150 miles in 2 hours. Use the MVT to explain why the speedometer must have read exactly 75 mph at least once during the trip. What assumption are you making about the car's position function?

2. Two toll booths on a highway are 60 miles apart. A driver passes the first booth at 2:00 PM and the second at 2:45 PM. The speed limit is 70 mph. The driver receives a speeding ticket at the second booth. Use the MVT to justify the ticket.

3. A trail runner completes a 27-mile ultramarathon in exactly 3 hours. Show that at some moment the runner was moving at exactly 9 mph.

4. A particle's position is $s(t) = t^3 - 6t^2 + 9t$ on $[0, 4]$. Find every time t where the instantaneous velocity equals the average velocity on $[0, 4]$.

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8. AP-Style Problem: MVT with a Table

A runner's velocity $v(t)$, in meters per second, is differentiable. Selected values are shown.

t (sec)	0	2	5	7	10
$v(t)$ (m/sec)	0	12	20	18	8

1. Must there be a time t in $(2, 5)$ at which the acceleration $a(t) = v'(t)$ is $\frac{8}{3}$ m/sec²? Justify your answer.

2. Must there be a time t in $(5, 10)$ at which $a(t) = -2.4$ m/sec²? Justify.

3. Must there be a time t in $(0, 10)$ at which $v(t) = 15$ m/sec? Name the theorem you use. (Hint: this is not the MVT.)

4. Is there a time in $(5, 7)$ at which $a(t) = 0$ must occur? Explain why the MVT alone does not decide this.

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9. Corollaries of the MVT

1. If $f'(x) > 0$ on an interval, then f is **increasing** there. If $f'(x) < 0$, then f is **decreasing**.
2. If $f'(x) = 0$ at every point of an interval, then f is **constant** on that interval.
3. If $f'(x) = g'(x)$ on an interval, then $f(x) = g(x) + C$ for some constant C .

10. Increasing and Decreasing Intervals

Find the intervals on which each function is increasing and decreasing. Show a sign chart for f' .

1.

$$f(x) = x^3 - 12x$$

2.

$$g(x) = xe^{-x}$$

3.

$$h(x) = x - 2\sin x, \quad 0 \leq x \leq 2\pi$$

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11. Functions with the Same Derivative

Use Corollary 3: find the function that satisfies the given conditions.

1.

$$f'(x) = 2x + 3, \quad f(1) = 6$$

2.

$$f'(x) = \cos x, \quad f(0) = 2$$

3.

$$f'(x) = e^x, \quad \text{the graph passes through } (0, 5)$$

4. An object falls with velocity $v(t) = 9.8t + 5$ m/sec. Its position at $t = 0$ is $s(0) = 10$ m. Find the position function $s(t)$.

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12. Prove It with the MVT

1. Use the MVT to prove Corollary 2: if $f'(x) = 0$ for every x in (a, b) , then f is constant on $[a, b]$. (Hint: pick any $x_1 < x_2$ in the interval.)

2. Use the MVT to show that $|\sin a - \sin b| \leq |a - b|$ for all real numbers a and b .

3. Show that $x^3 + x + 1 = 0$ has *exactly one* real solution. (Use the Intermediate Value Theorem to show there is at least one, and Rolle's Theorem to show there cannot be two.)

13. Explain Your Thinking

Why does the MVT require f to be differentiable on the *open* interval (a, b) but only continuous on the *closed* interval $[a, b]$?

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MEAN VALUE THEOREM

Mixed Review

14. Mixed Practice

Show your work clearly.

1. Find the value of c guaranteed by the MVT for $f(x) = x^2 - 3x + 1$ on $[1, 4]$.

2. Find every c guaranteed by Rolle's Theorem for $f(x) = \cos(2x)$ on $[0, \pi]$.

3. Explain why the MVT does not apply to $f(x) = \tan x$ on $[0, \pi]$.

4. Find $f(x)$ if $f'(x) = 3x^2 - 4x$ and $f(2) = 1$.

15. Challenge Problem

Suppose f is differentiable for all x , $f(0) = -3$, and $f'(x) \leq 5$ for every x . Use the MVT to find the largest possible value of $f(2)$.

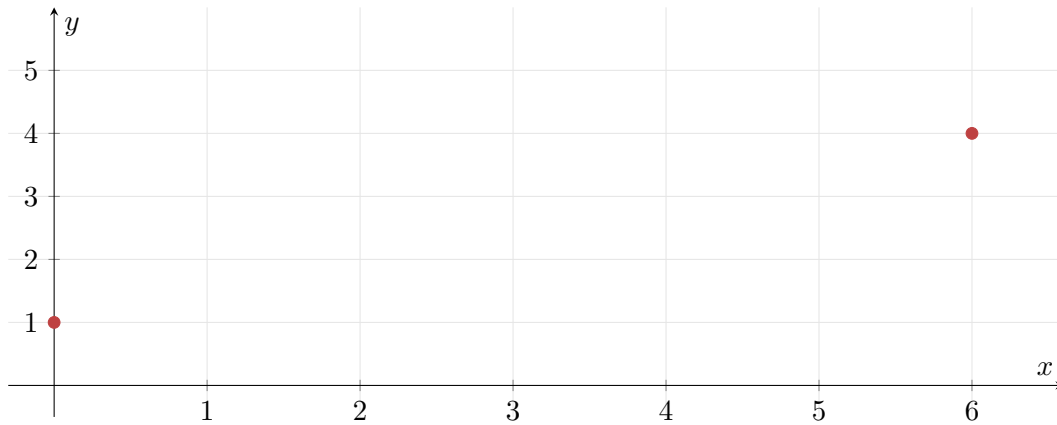
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16. Graphing Challenge

Sketch the graph of a function f on $[0, 6]$ that satisfies all of the following:

- f is continuous on $[0, 6]$ and differentiable on $(0, 6)$
- $f(0) = 1$ and $f(6) = 4$
- There are **exactly three** values of c in $(0, 6)$ that satisfy the conclusion of the MVT



Draw the secant line and the three parallel tangent lines. Then explain why a function *cannot* satisfy these conditions with zero values of c .

Reflection: What is one thing you understand better about the Mean Value Theorem after completing this worksheet?
