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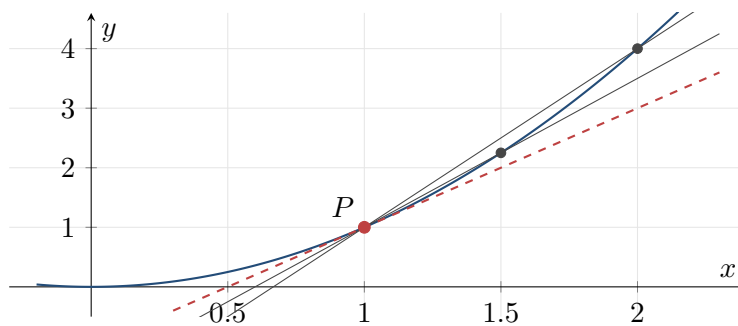
RATES OF CHANGE AND TANGENT LINES

AP Calculus AB Practice Worksheet

Learning Goal: Use secant lines and limits to find the slope of a curve at a point, write equations of tangent and normal lines, locate points with a given slope, and interpret slopes as instantaneous rates of change.

1. From Secant Lines to a Tangent Line

Let $f(x) = x^2$ and $P = (1, 1)$. For each point $Q = (x, f(x))$, find the slope of the secant line PQ .



x	2	1.5	1.1	1.01	0	0.5	0.9	0.99
Slope PQ								

- What value do the secant slopes approach as $Q \rightarrow P$ from either side? _____
- Write an equation of the tangent line to $y = x^2$ at P .

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2. Slope of a Curve at a Point

The **slope of the curve** $y = f(x)$ at $x = a$ is

$$m = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

provided the limit exists. The **tangent line** at $(a, f(a))$ is the line through that point with slope m :

$$y - f(a) = m(x - a)$$

The **normal line** is perpendicular to the tangent line, so its slope is $-\frac{1}{m}$ (when $m \neq 0$).

3. Find the Slope Using the Limit

Use the limit definition. Show every algebraic step.

1.

$$f(x) = x^2 - 3x, \quad a = 2$$

2.

$$f(x) = 5 - 2x^2, \quad a = -1$$

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4. More Slopes Using the Limit

Use the limit definition to find the slope of each curve at the given point.

1.

$$f(x) = \frac{4}{x}, \quad a = -2 \quad (\text{Hint: combine fractions in the numerator.})$$

2.

$$f(x) = \sqrt{x+1}, \quad a = 3 \quad (\text{Hint: rationalize the numerator.})$$

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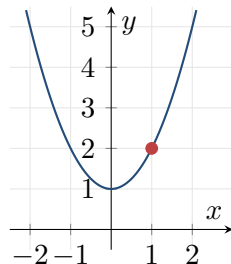
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5. Equations of Tangent and Normal Lines

For each curve, find the slope at the given point using a limit. Then write the equations of the tangent line *and* the normal line.

1.

$$y = x^2 + 1, \quad \text{at } (1, 2)$$



Sketch the tangent and normal lines on the graph.

2.

$$y = \frac{2}{x}, \quad \text{at } x = 1$$

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6. Slope at Any Point

Use a general point x instead of a number a .

1. Use the limit definition to show that the slope of $y = x^2 - 6x + 5$ at any point x is $2x - 6$.

2. Use your formula to find the slope at $x = 0$, $x = 3$, and $x = 5$. Where is the tangent line horizontal?

3. Use the limit definition to find the slope of $y = x^3 - 3x$ at any point x . (Hint: $(x + h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$.)

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7. Horizontal Tangents and Given Slopes

1. Using your result from Section 6, find all points on $y = x^3 - 3x$ where the tangent line is horizontal.

2. Find the point on the curve $y = x^2$ where the tangent line is parallel to the line $y = 4x - 1$.

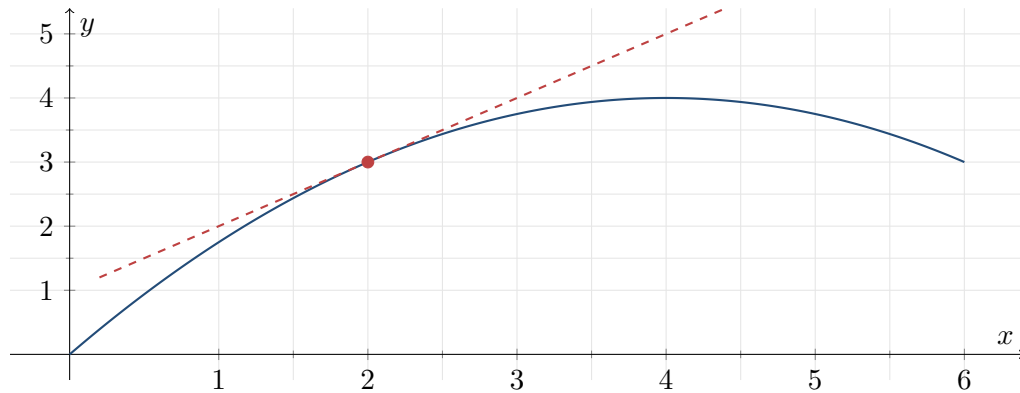
3. Find the point on the curve $y = x^2 + x$ where the tangent line is perpendicular to the line $x + 3y = 6$.

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8. Estimating Slopes from a Graph

The graph of $y = f(x)$ is shown with a tangent line (dashed) drawn at $x = 2$.



1. Use two points on the dashed tangent line to find the slope of the curve at $x = 2$.

2. Estimate the slope of the curve at $x = 4$ and at $x = 5$. Draw the tangent lines you used.

3. Where on the interval $(0, 6)$ is the slope of the curve positive? Negative? Zero?

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9. Estimating Slopes from Data

The table gives the temperature T , in $^{\circ}\text{C}$, of a cup of tea t minutes after it is poured.

t (min)	0	2	4	6	8	10
$T(t)$ ($^{\circ}\text{C}$)	90	78	69	62	57	53

1. Find the average rate of change of the temperature from $t = 0$ to $t = 10$. Include units.

2. Estimate the instantaneous rate of change at $t = 4$ using the symmetric difference quotient $\frac{T(6) - T(2)}{6 - 2}$. Interpret your answer in context.

3. Is the tea cooling faster at $t = 2$ or at $t = 8$? Justify with estimates.

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10. Instantaneous Rates in Motion

1. A stone is dropped from a bridge. It falls $y = 4.9t^2$ meters in t seconds. Use a limit to find the stone's speed at $t = 2$ seconds.

2. A particle moves along a line with position $s(t) = t^2 - 5t + 6$ meters.

(a) Use a limit to find the instantaneous velocity at any time t .

(b) When is the particle at rest? Where is it at that time?

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11. When a Tangent Line Does Not Exist

1. For $f(x) = |x|$ at $x = 0$, compute

$$\lim_{h \rightarrow 0^-} \frac{f(0+h) - f(0)}{h} \quad \text{and} \quad \lim_{h \rightarrow 0^+} \frac{f(0+h) - f(0)}{h}.$$

Does the curve have a tangent line at the origin? Explain.

2. For $f(x) = x^{1/3}$ at $x = 0$, compute $\lim_{h \rightarrow 0} \frac{f(h) - f(0)}{h}$. What does this tell you about the tangent line at the origin?

12. Explain Your Thinking

1. Explain why the slope of a secant line approaches the slope of the tangent line as the second point moves toward the point of tangency.

2. What is the relationship between the slope of a curve and the instantaneous rate of change of the function?

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13. AP-Style ProblemLet $f(x) = x^2 - 4x$.

1. Find the average rate of change of f on the interval $[1, 4]$.

2. Use the limit definition to find the instantaneous rate of change of f at any point x .

3. Find the value of c in $(1, 4)$ where the instantaneous rate of change equals the average rate of change from part (a).

4. Write the equation of the tangent line at $x = c$. Explain why it is parallel to the secant line through $(1, f(1))$ and $(4, f(4))$.

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Mixed Review

14. Mixed Practice

Show your work clearly.

1. Use a limit to find the slope of $y = 3x^2 - x$ at $x = 2$.

2. Write the equations of the tangent and normal lines to $y = x^2 - 2x$ at $x = 3$.

3. Find the point on $y = x^2 - 4x + 1$ where the tangent line is horizontal.

15. Challenge Problem

Find the equations of *both* lines that pass through the point $(1, -3)$ and are tangent to the parabola $y = x^2$. (Hint: let the point of tangency be (a, a^2) .)

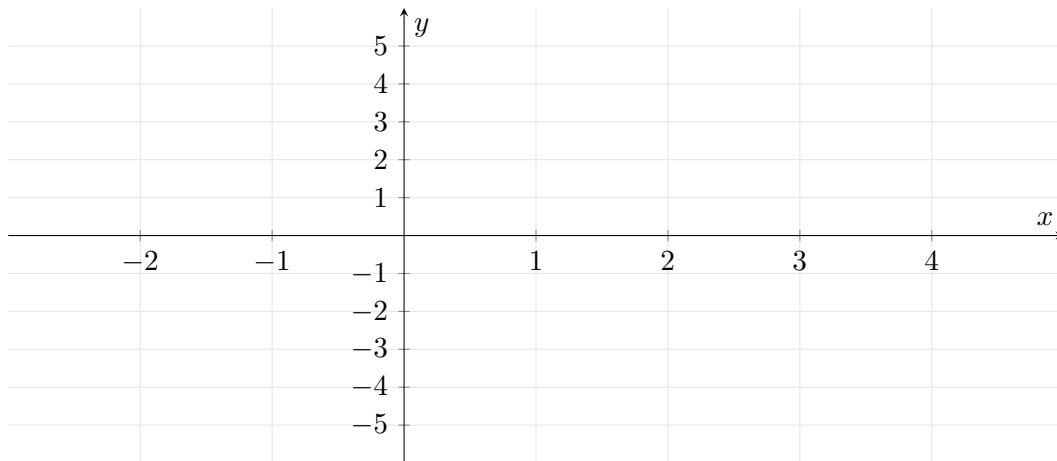
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16. Graphing Challenge

On the grid, graph $y = x^2 - 2x - 3$. Then:

- Draw the tangent lines at $x = -1$, $x = 1$, and $x = 3$.
- Label each tangent line with its slope (found using a limit or your slope formula).
- Draw the normal line at $x = 3$.



1. Slopes: at $x = -1$: _____ at $x = 1$: _____ at $x = 3$: _____

2. What do you notice about the tangent lines at $x = -1$ and $x = 3$? Explain why using symmetry.

Reflection: What is one thing you understand better about rates of change and tangent lines after completing this worksheet?
