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RATES OF CHANGE

AP Calculus AB Practice Worksheet

Learning Goal: Compute average rates of change from formulas, graphs, and tables, use limits to find instantaneous rates of change, write equations of tangent and normal lines, and interpret rates of change in context with correct units.

1. Average and Instantaneous Speed

A rock is dropped from a cliff. After t seconds it has fallen $y = 16t^2$ feet.

1. Find the average speed of the rock during the first 2 seconds.

2. Find the average speed between $t = 1$ and $t = 2$.

3. Complete the table of average speeds on the interval $[2, 2 + h]$.

h	1	0.1	0.01	0.001	0.0001
$\frac{16(2+h)^2 - 16(2)^2}{h}$					

Instantaneous speed at $t = 2 \approx$ _____ ft/sec

4. Simplify $\frac{16(2+h)^2 - 64}{h}$ algebraically and confirm your estimate.

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2. Average Rate of Change

The **average rate of change** of $y = f(x)$ over $[a, b]$ is

$$\frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}$$

Geometrically, it is the slope of the **secant line** through $(a, f(a))$ and $(b, f(b))$.

3. Compute Average Rates of Change

Find the average rate of change of each function over the given interval.

1. $f(x) = x^3 + 1$, $[2, 3]$

2. $f(x) = \sqrt{4x + 1}$, $[0, 2]$

3. $f(x) = e^x$, $[-2, 0]$

4. $f(x) = \cos x$, $[0, \pi]$

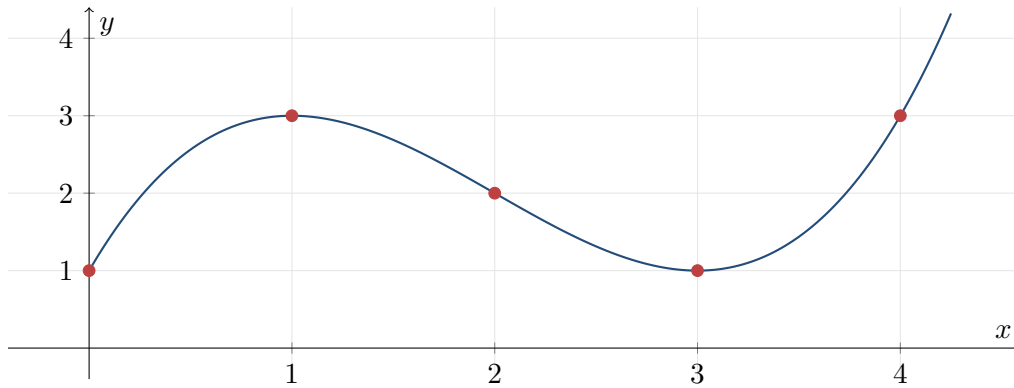
5. $f(x) = \frac{1}{x}$, $[a, a + h]$ (simplify your answer)

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4. Average Rates of Change from a Graph

The graph of $y = f(x)$ is shown. The marked points have integer coordinates.



1. Find the average rate of change of f over each interval.

Interval	[0, 1]	[1, 3]	[0, 4]	[2, 4]	[0, 3]
Avg. rate					

2. On which interval in the table is f changing fastest on average? On which is the average rate 0?

3. Draw the secant line for $[1, 3]$ on the graph. Estimate a point in $(1, 3)$ where the tangent line is parallel to it.

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5. Rates of Change from a Table

The population $P(t)$ of a town, in thousands, is shown for selected years, where t is years since 2000.

t (years)	0	5	10	15	20
$P(t)$ (thousands)	12.0	13.5	15.4	17.8	20.6

1. Find the average rate of change of the population from $t = 0$ to $t = 20$. Include units, and write a sentence explaining its meaning.

2. Complete the table of average rates of change.

Interval	$[0, 5]$	$[5, 10]$	$[10, 15]$	$[15, 20]$
Avg. rate (thousand/yr)				

3. Is the population growing at a constant rate, a faster rate, or a slower rate over time? Explain using your table.

4. Estimate the *instantaneous* rate of growth in the year 2010 ($t = 10$) by averaging the rates on $[5, 10]$ and $[10, 15]$.

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6. Instantaneous Rate of Change

The **instantaneous rate of change** of f at $x = a$ is

$$\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

provided the limit exists. Geometrically, it is the **slope of the tangent line** (also called the *slope of the curve*) at $(a, f(a))$.

7. Find the Slope of a Curve

Use the limit above to find the slope of each curve at the given point.

1. $f(x) = x^2$, $a = 3$

2. $f(x) = 2x^2 - 5x$, $a = 1$

3. $f(x) = \frac{1}{x}$, $a = 2$

4. $f(x) = \sqrt{x}$, $a = 4$ (Hint: rationalize the numerator.)

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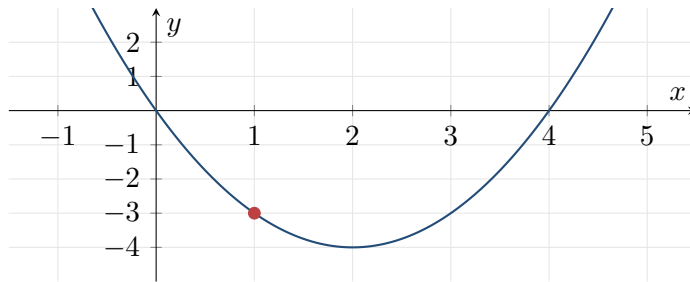
8. Tangent and Normal Lines

If m is the slope of the curve at $(a, f(a))$:

Tangent: $y - f(a) = m(x - a)$	Normal: $y - f(a) = -\frac{1}{m}(x - a)$
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The normal line is perpendicular to the tangent line at the point of tangency.

1. Find the equations of the tangent and normal lines to $y = x^2 - 4x$ at $x = 1$.



Sketch both lines on the graph.

2. Find the equation of the tangent line to $y = \frac{1}{x-1}$ at $x = 3$.

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9. A General Slope Formula

Instead of a number a , use a general point x .

1. Show that the slope of $y = x^2 + 4x - 1$ at any point x is $2x + 4$.

2. Use your formula to find where the curve has a horizontal tangent.

3. Find the slope of $y = x^3$ at any point x . (Hint: $(x + h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$.)

4. Find every point on $y = x^3$ where the slope is 12.

5. Is the slope of $y = x^3$ ever negative? Explain using your formula.

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10. Velocity as a Rate of ChangeIf $s(t)$ is position, then

Average velocity = $\frac{s(b) - s(a)}{b - a}$	Instantaneous velocity = $\lim_{h \rightarrow 0} \frac{s(t+h) - s(t)}{h}$
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1. A ball is thrown upward. Its height in feet after t seconds is

$$s(t) = 48t - 16t^2.$$

- (a) Find the average velocity on $[0, 1]$ and on $[1, 2]$.

- (b) Find the instantaneous velocity at $t = 1$ using the limit.

- (c) At what time is the instantaneous velocity 0? What is happening to the ball then?

2. A particle moves so that $s(t) = t^2 - 4t + 3$. Find its instantaneous velocity at $t = 3$. Is it moving forward or backward?

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11. Interpreting Rates in Context

Write a sentence that interprets each rate. Include units.

1. The temperature T (in $^{\circ}\text{F}$) of a cup of coffee t minutes after it is poured satisfies $T(10) = 150$ and $T(20) = 126$. Find and interpret $\frac{T(20) - T(10)}{20 - 10}$.

2. The cost $C(x)$, in dollars, of producing x shirts satisfies $\lim_{h \rightarrow 0} \frac{C(100 + h) - C(100)}{h} = 4.5$. Interpret this value.

3. Water drains from a tank. The volume (in gallons) after t minutes is $V(t) = 200 - 6t$. What is the rate of change of V ? What does its sign mean?

12. Explain Your Thinking

1. What is the difference between an average rate of change and an instantaneous rate of change? Use a car trip as an example.

2. For a linear function, why is the average rate of change the same on every interval?

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Mixed Review

13. Mixed Practice

Show your work clearly.

1. Find the average rate of change of $f(x) = x^2 - 3x$ on $[1, 4]$.

2. Use the limit definition to find the slope of $y = 3x^2 + 1$ at $x = -1$.

3. Write the equation of the normal line to $y = x^2$ at $(2, 4)$.

4. The height of a rocket is $s(t) = 5t^2$ meters. Find its instantaneous velocity at $t = 4$ seconds.

14. Challenge Problem

For $f(x) = x^2$, show that the average rate of change on *any* interval $[a, b]$ equals the instantaneous rate of change at the midpoint $\frac{a+b}{2}$.

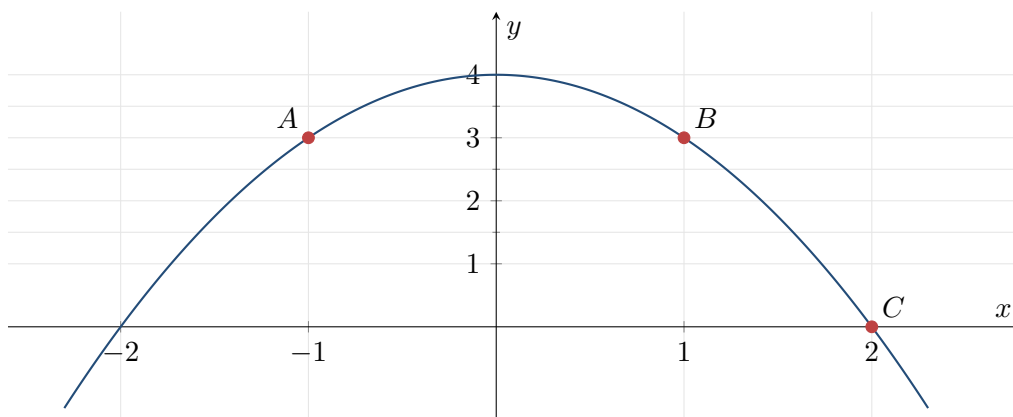
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15. Graphing Challenge

The graph of $f(x) = 4 - x^2$ is shown with three points marked.

- Draw the secant line from A to C and find its slope.
- Draw the tangent line at B and find its slope using a limit.
- Draw secant lines from B to points closer and closer to B . What do you notice?



1. Slope of secant AC = _____ Slope of tangent at B = _____

2. Explain how the secant slopes from B relate to the tangent slope at B .

Reflection: What is one thing you understand better about rates of change after completing this worksheet?
