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RELATED RATES

AP Calculus AB Practice Worksheet

Learning Goal: Model situations in which several quantities change with time, differentiate equations implicitly with respect to time, and solve for an unknown rate using known rates and values, with correct units and interpretation.

1. Strategy for Related Rates Problems

1. **Draw** a picture. Label quantities that change with *variables* and quantities that stay fixed with *numbers*.
2. **List** the given rates and the rate you need, using $\frac{d}{dt}$ notation.
3. **Write an equation** relating the variables (geometry, similar triangles, Pythagorean Theorem, trig).
4. **Differentiate** both sides with respect to t (chain rule!).
5. **Substitute** the known values *after* differentiating, then solve.
6. **Interpret** your answer with units. A negative rate means decreasing.

2. Warm-Up: Differentiate with Respect to Time

Every variable below is a function of t . Differentiate both sides with respect to t .

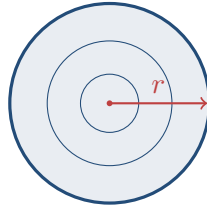
Equation	Differentiated with respect to t
$A = \pi r^2$	
$V = \frac{4}{3}\pi r^3$	
$x^2 + y^2 = z^2$	
$A = \frac{1}{2}bh$ (b and h both vary)	
$\tan \theta = \frac{y}{50}$	

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3. Circles and Spheres

1. A stone dropped in a pond makes a circular ripple whose radius grows at 2 ft/sec. How fast is the area inside the ripple increasing when the radius is 10 ft?



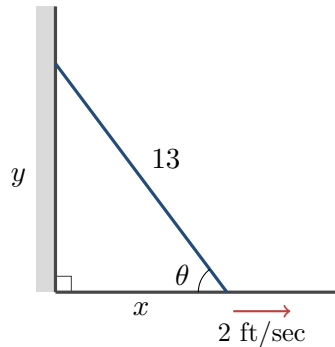
2. Air is pumped into a spherical balloon at $100 \text{ cm}^3/\text{sec}$. How fast is the radius increasing when the radius is 5 cm?

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4. The Sliding Ladder

A 13-ft ladder leans against a vertical wall. The bottom of the ladder slides away from the wall at 2 ft/sec.



1. How fast is the top of the ladder sliding down the wall when the bottom is 5 ft from the wall?

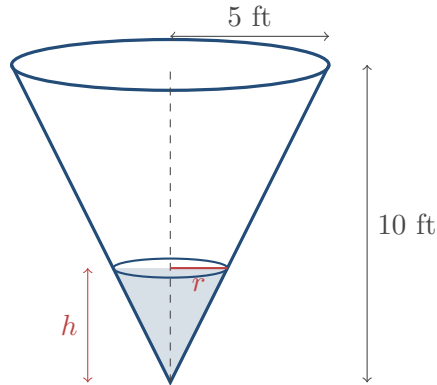
2. How fast is the angle θ between the ladder and the ground changing at that moment?

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5. Filling a Conical Tank

Water is poured into an inverted conical tank at $9 \text{ ft}^3/\text{min}$. The tank is 10 ft tall and has a radius of 5 ft at the top.



1. Use similar triangles to write r in terms of h . Then write the volume of water V as a function of h alone.
($V = \frac{1}{3}\pi r^2 h$)

2. How fast is the water level rising when the water is 6 ft deep?

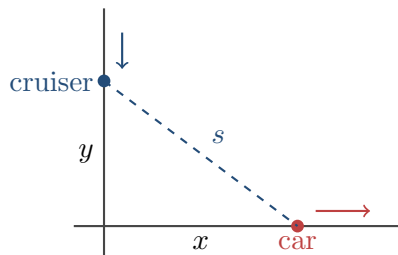
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6. Moving Objects

1. Two cars leave an intersection at the same time. One drives north at 60 mph, and the other drives east at 45 mph. How fast is the distance between them increasing after 2 hours?

2. A police cruiser approaches a right-angled intersection from the north at 60 mph. A car is moving east from the intersection. When the cruiser is 0.6 mi north of the intersection and the car is 0.8 mi east, the radar shows that the distance between them is *increasing* at 20 mph. How fast is the car moving?

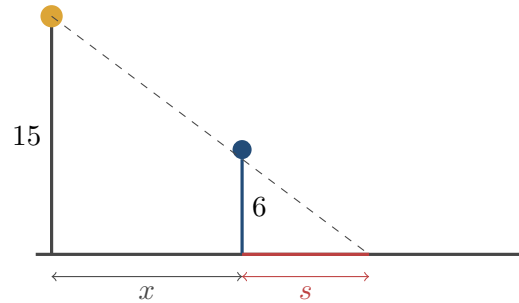


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7. Shadows and Similar Triangles

A 6-ft-tall person walks away from a 15-ft streetlight at 5 ft/sec.



1. Use similar triangles to write the shadow length s in terms of x .

2. How fast is the length of the shadow increasing?

3. How fast is the tip of the shadow moving away from the streetlight?

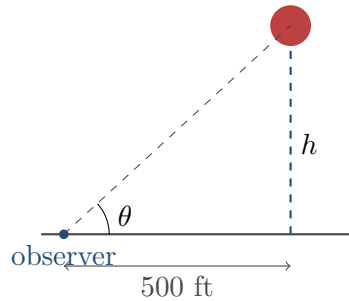
4. Do your answers depend on how far the person is from the light? Explain.

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8. Rates Involving Angles

1. A hot-air balloon rises straight up from a field. An observer 500 ft from the lift-off point measures the angle of elevation. At the moment the angle is $\frac{\pi}{4}$, it is increasing at 0.14 rad/min. How fast is the balloon rising?



2. A kite flies at a constant height of 100 ft. The wind carries it horizontally away from the person flying it at 8 ft/sec. How fast must the string be let out when 200 ft of string is out?

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9. Changing Shapes

1. The length of a rectangle increases at 3 cm/sec while its width decreases at 2 cm/sec. When the length is 10 cm and the width is 6 cm, find the rates of change of

(a) the area,

(b) the perimeter,

(c) the length of a diagonal.

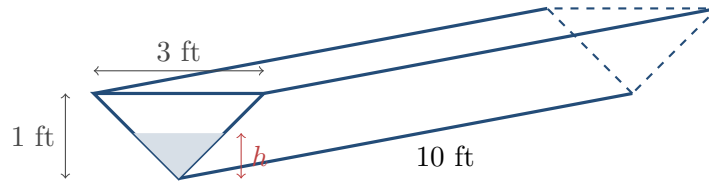
2. The edges of a cube are expanding at 0.5 cm/sec. How fast is the volume changing when each edge is 4 cm? How fast is the surface area changing?

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10. AP-Style Problem: The Water Trough

A water trough is 10 ft long. Its ends are isosceles triangles that are 3 ft across at the top and 1 ft deep. Water is pumped into the trough at $12 \text{ ft}^3/\text{min}$.



1. Show that when the water is h ft deep, the width of the water surface is $3h$ ft, and the volume of water is $V = 15h^2$.

2. How fast is the water level rising when the water is 6 inches deep?

3. How fast is the width of the water surface changing at that moment?

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11. Find the Error

A student solves the ripple problem from Section 3 like this:

“When $r = 10$, the area is $A = \pi(10)^2 = 100\pi$. Then $\frac{dA}{dt} = \frac{d}{dt}(100\pi) = 0$. So the area is not changing.”

1. Explain the mistake.

2. Write a correct solution.

12. Explain Your Thinking

1. In the ladder problem, $\frac{dy}{dt}$ is negative. What does the sign tell you about the ladder?

2. In the shadow problem, why can the answer be found without knowing the person's distance from the light, while in the ladder problem it cannot?

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RELATED RATES

Mixed Review

13. Mixed Practice

Show your work clearly and include units.

1. The side of a square increases at 4 in/min. How fast is the area increasing when the area is 81 in²?

2. A particle moves along the curve $y = \sqrt{x}$. Its x -coordinate increases at 3 units/sec. How fast is its distance from the origin changing at the point (4, 2)?

3. The radius of a circular oil spill grows at 0.5 m/min. How fast is the circumference changing when the radius is 20 m?

14. Challenge Problem

A spherical snowball melts so that its volume decreases at a rate proportional to its surface area: $\frac{dV}{dt} = -kS$ for some constant $k > 0$. Show that its radius decreases at a constant rate.

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15. Diagram Challenge

A boat is pulled toward a dock by a rope attached to its bow. The rope runs through a pulley on the dock that is 6 ft higher than the bow. The rope is pulled in at 2 ft/sec.

- Draw and label a diagram in the box below.
- How fast is the boat approaching the dock when 8 ft of rope is out?

Does the boat move faster or slower than the rope is being pulled in? Explain.

Reflection: What is one thing you understand better about related rates after completing this worksheet?