

Name: \_\_\_\_\_

Date: \_\_\_\_\_

# RULES OF DIFFERENTIATION

## AP Calculus AB Practice Worksheet

**Learning Goal:** Differentiate functions efficiently using the constant, power, constant multiple, sum and difference, product, and quotient rules; find higher-order derivatives; and apply the rules to tables, graphs, tangent lines, and rates of change.

### 1. Basic Rules

|                                 |                                                           |
|---------------------------------|-----------------------------------------------------------|
| <b>Constant Rule:</b>           | $\frac{d}{dx}(c) = 0$                                     |
| <b>Power Rule:</b>              | $\frac{d}{dx}(x^n) = nx^{n-1}$ (any integer $n$ )         |
| <b>Constant Multiple Rule:</b>  | $\frac{d}{dx}(cu) = c \frac{du}{dx}$                      |
| <b>Sum and Difference Rule:</b> | $\frac{d}{dx}(u \pm v) = \frac{du}{dx} \pm \frac{dv}{dx}$ |

### 2. Warm-Up: Rewrite, Then Differentiate

Rewrite each function using exponents if needed. Then find the derivative.

| Function             | Rewrite | Derivative |
|----------------------|---------|------------|
| $y = x^9$            |         |            |
| $y = -7$             |         |            |
| $y = \frac{1}{x^4}$  |         |            |
| $y = \frac{5}{x}$    |         |            |
| $y = \frac{x^3}{6}$  |         |            |
| $y = \frac{3}{2x^2}$ |         |            |

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**3. Power, Sum, and Difference Rules**Find  $\frac{dy}{dx}$ .

1.

$$y = x^5 - 4x^3 + 2x - 9$$

2.

$$y = \frac{x^4}{2} - \frac{2x^3}{3} + x$$

3.

$$y = (2x + 1)^2 \quad (\text{Hint: expand first.})$$

4.

$$y = 4x^{-2} - \frac{3}{x^3} + \frac{1}{x}$$

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**4. The Product Rule**

$$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$$

“First times the derivative of the second, plus second times the derivative of the first.”

Use the Product Rule to find  $\frac{dy}{dx}$ . Do not expand first.

1.

$$y = (x^2 + 1)(x^3 + 3)$$

2.

$$y = (2x - 1)(x^2 - 4x + 5)$$

3. Check your answer to problem 1 by expanding first and then differentiating. Do you get the same result?

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## 5. The Quotient Rule

$$\frac{d}{dx} \left( \frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}, \quad v \neq 0$$

“Low d-high minus high d-low, over low squared.”

Use the Quotient Rule to find  $\frac{dy}{dx}$ . Simplify.

1.

$$y = \frac{x^2 - 1}{x^2 + 1}$$

2.

$$y = \frac{2x + 5}{3x - 2}$$

3.

$$y = \frac{x^3}{1 - x}$$

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**6. Choose the Best Method**

Sometimes simplifying first is faster than using the Product or Quotient Rule. Decide which method to use, then find  $\frac{dy}{dx}$ .

1.

$$y = \frac{x^4 - 3x^2 + 5}{x}$$

2.

$$y = \frac{x^2 + 3}{x}$$

3.

$$y = \left(x + \frac{1}{x}\right) \left(x - \frac{1}{x} + 1\right)$$

4.

$$y = \frac{(x-1)(x^2+x+1)}{x^3}$$

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**7. Second and Higher-Order Derivatives**

$$y'' = \frac{d^2y}{dx^2} = \frac{d}{dx} \left( \frac{dy}{dx} \right), \quad y''' = \frac{d^3y}{dx^3}, \quad y^{(n)} = \frac{d^ny}{dx^n}$$

1. Find all derivatives of  $y = x^4 - 3x^3 + 2x$  until you reach 0.

|           |  |
|-----------|--|
| $y'$      |  |
| $y''$     |  |
| $y'''$    |  |
| $y^{(4)}$ |  |
| $y^{(5)}$ |  |

2. Find  $y''$  for  $y = \frac{1}{x}$ . Then find  $y'''$  and  $y^{(4)}$ . Describe the pattern and write a formula for  $y^{(n)}$ .

3. Find  $\frac{d^2y}{dx^2}$  for  $y = \frac{x}{x+1}$ .

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**8. AP-Style Problem: Derivatives from a Table**

The functions  $f$  and  $g$  are differentiable. Selected values are shown.

| $x$ | $f(x)$ | $f'(x)$ | $g(x)$ | $g'(x)$ |
|-----|--------|---------|--------|---------|
| 1   | 3      | 2       | 1      | -3      |
| 2   | -1     | 4       | 5      | 2       |
| 3   | 2      | -1      | -2     | 6       |

Find each value. Show the rule you use.

1.  $h'(2)$ , where  $h(x) = f(x)g(x)$

2.  $q'(2)$ , where  $q(x) = \frac{f(x)}{g(x)}$

3.  $k'(1)$ , where  $k(x) = 3f(x) - 2g(x) + x^2$

4.  $m'(3)$ , where  $m(x) = x^2f(x)$

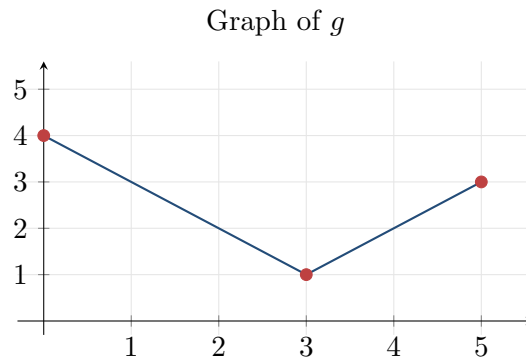
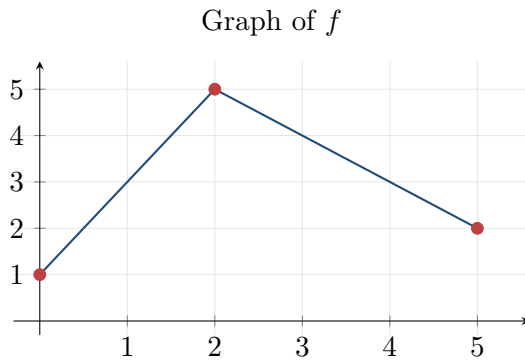
5.  $p'(1)$ , where  $p(x) = \frac{f(x) + 1}{xg(x)}$

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**9. Derivatives from Graphs**

The graphs of  $f$  and  $g$  consist of line segments.



1. Let  $h(x) = f(x)g(x)$ . Find  $h'(1)$ .

2. Let  $q(x) = \frac{f(x)}{g(x)}$ . Find  $q'(4)$ .

3. Let  $s(x) = 2f(x) - g(x)$ . Find  $s'(4)$ .

4. Explain why  $h'(2)$  does not exist.

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**10. Tangent Lines and Horizontal Tangents**

1. Find all points on the graph of  $y = x^3 - 6x^2 + 9x$  where the tangent line is horizontal.

2. Find all points on the graph of  $y = \frac{x}{x^2 + 1}$  where the tangent line is horizontal.

3. Write the equation of the tangent line to  $y = \frac{x^2 + 4}{x}$  at  $x = 1$ .

4. Find every point on  $y = x^3 - 3x + 1$  where the slope of the tangent line is 9.

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**11. Rates of Change in Context**

1. The cost of producing  $x$  units is  $C(x) = 0.02x^3 - 3x^2 + 200x + 500$  dollars. Find the marginal cost  $C'(x)$  and evaluate  $C'(50)$ . Interpret your answer.

2. A population of bacteria is modeled by  $P(t) = \frac{500t}{t+4}$ , where  $t$  is in hours. Find  $P'(t)$  and  $P'(6)$ . What does  $P'(6)$  mean?

3. A particle moves with position  $s(t) = t^3 - 9t^2 + 24t$  meters. Find its velocity  $v(t)$  and acceleration  $a(t)$ . When is the particle at rest?

**12. Explain Your Thinking**

A student claims that  $\frac{d}{dx}[f(x)g(x)] = f'(x)g'(x)$ . Use  $f(x) = x$  and  $g(x) = x^2$  to show that this is false.

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**RULES OF DIFFERENTIATION**

## Mixed Review

**13. Mixed Practice**

Show your work clearly.

1. Find  $\frac{dy}{dx}$  for  $y = (3x^2 - 1)(x^{-1} + 2)$ .

2. Find  $\frac{dy}{dx}$  for  $y = \frac{x^2}{x^2 - 9}$ .

3. Find  $y''$  for  $y = 6x^5 - \frac{2}{x^2}$ .

**14. Challenge Problem**

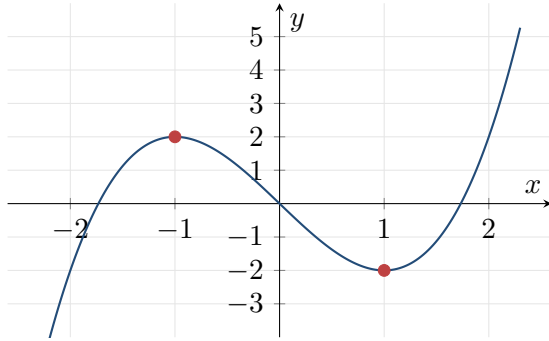
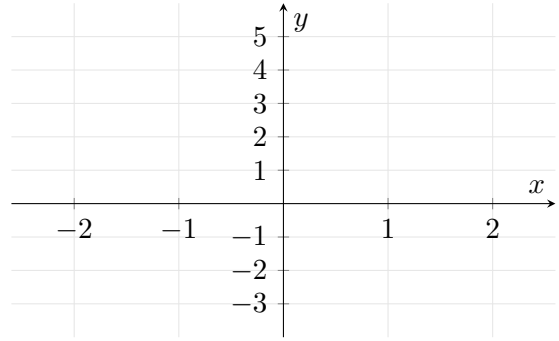
Find  $a$ ,  $b$ , and  $c$  so that the parabola  $y = ax^2 + bx + c$  passes through  $(0, 1)$  and is tangent to the line  $y = x$  at the point  $(1, 1)$ .

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## 15. Graphing Challenge

The graph of  $f(x) = x^3 - 3x$  is shown. On the second grid, sketch the graph of  $f'$ . Use your derivative rules to check key points.

Graph of  $f$ Sketch  $f'$  here

1. Find  $f'(x)$  and  $f''(x)$ .

2. Where is  $f'(x) = 0$ ? How does that show up on each graph?

**Reflection:** What is one thing you understand better about the rules of differentiation after completing this worksheet?