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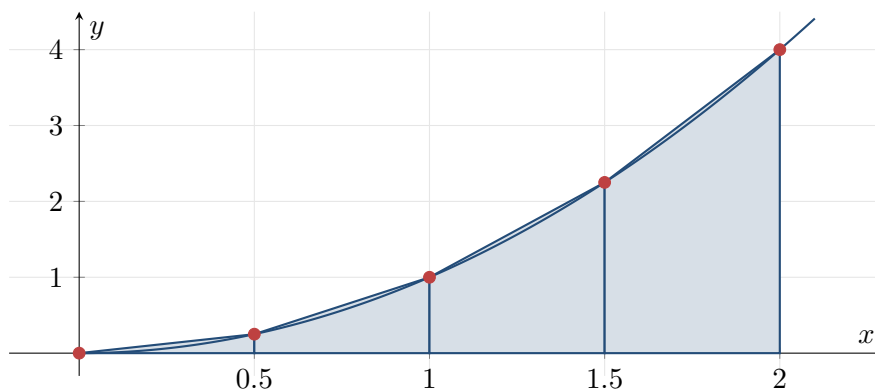
TRAPEZOIDAL RULE

AP Calculus AB Practice Worksheet

Learning Goal: Approximate definite integrals with trapezoids, apply the Trapezoidal Rule to formulas and to tables with equal or unequal subintervals, decide whether a trapezoidal sum overestimates or underestimates, and interpret results in context.

1. Why Trapezoids?

The graph shows $f(x) = x^2$ on $[0, 2]$ divided into 4 subintervals. Each strip is a trapezoid whose top edge connects two points on the curve.



1. The area of a trapezoid with parallel sides y_1 and y_2 and width h is $\frac{h}{2}(y_1 + y_2)$. Find the area of each trapezoid.

Interval	$[0, 0.5]$	$[0.5, 1]$	$[1, 1.5]$	$[1.5, 2]$
Area				

2. Add the four areas. Compare your total with the exact area $\frac{8}{3} \approx 2.667$.

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2. The Trapezoidal Rule

To approximate $\int_a^b f(x) dx$, divide $[a, b]$ into n subintervals of equal width $h = \frac{b-a}{n}$, and let $y_k = f(x_k)$. Then

$$T = \frac{h}{2} (y_0 + 2y_1 + 2y_2 + \cdots + 2y_{n-1} + y_n)$$

Notice that $T = \frac{\text{LRAM}_n + \text{RRAM}_n}{2}$. The endpoints are counted once; every interior point is counted twice.

3. Apply the Trapezoidal Rule

Use the Trapezoidal Rule with the given n . Then compare with the exact value.

1. $\int_0^2 x^3 dx$, $n = 4$. Exact value: 4.

x_k	0	0.5	1	1.5	2
$y_k = x_k^3$					

2. $\int_0^4 \sqrt{x} dx$, $n = 4$. Exact value: $\frac{16}{3}$.

3. $\int_1^3 \frac{1}{x} dx$, $n = 4$. Exact value: $\ln 3$.

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4. More Practice

Use the Trapezoidal Rule with the given n . Round to four decimal places.

1. $\int_0^{\pi} \sin x \, dx$, $n = 4$. Compare with the exact value 2.

2. $\int_0^1 e^x \, dx$, $n = 4$. Compare with the exact value $e - 1$.

3. $\int_{-1}^1 (1 - x^2) \, dx$, $n = 4$. Compare with the exact value $\frac{4}{3}$.

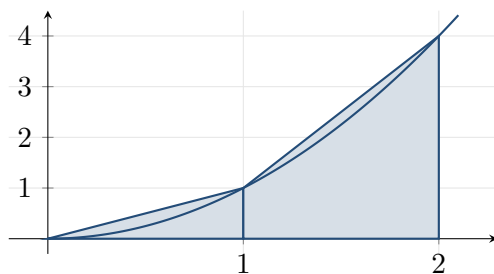
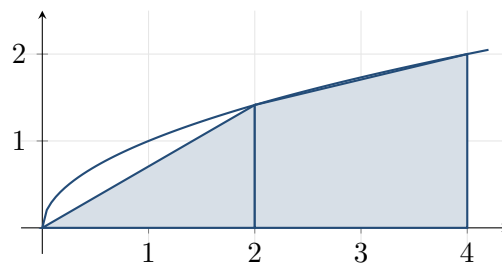
4. $\int_0^2 (2x + 1) \, dx$, $n = 2$. Compare with the exact value. Why is the Trapezoidal Rule exact here?

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5. Overestimate or Underestimate?

If f is ...	Trapezoid tops lie	T is an ...
concave up ($f'' > 0$)	above the curve	overestimate
concave down ($f'' < 0$)	below the curve	underestimate

Concave up: T too largeConcave down: T too smallDecide whether the Trapezoidal Rule gives an overestimate or an underestimate. Justify using f'' .

Integral	$f''(x)$	Concavity	Over/Under	Reason
$\int_0^2 x^3 dx$				
$\int_0^4 \sqrt{x} dx$				
$\int_1^3 \frac{1}{x} dx$				
$\int_0^\pi \sin x dx$				
$\int_0^1 e^x dx$				

Check each answer against your results from Sections 3 and 4.

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6. Trapezoids from a Table

A car's speed is recorded every 2 seconds.

t (sec)	0	2	4	6	8	10
$v(t)$ (ft/sec)	0	18	30	38	42	44

1. Use the Trapezoidal Rule with 5 subintervals to estimate the distance the car travels in the first 10 seconds.

2. Explain what $\int_0^{10} v(t) dt$ represents in this context. Include units.

7. Unequal Subintervals

When the widths are not equal, add the areas of the individual trapezoids:

$$\text{Area of each trapezoid} = \frac{(\text{width})}{2} (\text{left height} + \text{right height})$$

Water flows into a tank at $R(t)$ gallons per hour.

t (hours)	0	3	4	7	10
$R(t)$ (gal/hr)	10	14	16	12	8

1. Use a trapezoidal sum with the four subintervals shown to approximate $\int_0^{10} R(t) dt$. Show the computation.

2. Interpret your answer in context, with units.

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8. AP-Style Problem

A hiker's elevation changes at a rate of $E'(t)$ feet per hour, where t is measured in hours. The function E' is continuous, and selected values are shown.

t (hours)	0	1	3	4	6
$E'(t)$ (ft/hr)	400	520	300	-100	-250

1. Use a trapezoidal sum with the four subintervals shown to approximate $\int_0^6 E'(t) dt$.

2. Explain the meaning of $\int_0^6 E'(t) dt$ in the context of the problem.

3. The hiker starts at an elevation of 2000 ft. Use your answer from part (a) to estimate the hiker's elevation at $t = 6$.

4. Use a right Riemann sum with the same subintervals to approximate $\int_0^6 E'(t) dt$. Which estimate do you expect to be more accurate? Explain.

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9. Comparing Methods

For $\int_0^2 x^2 dx = \frac{8}{3} \approx 2.6667$, complete the table.

n	LRAM $_n$	RRAM $_n$	T_n	Error $ T_n - \frac{8}{3} $
4	1.75	3.75		
8	2.1875	3.1875		

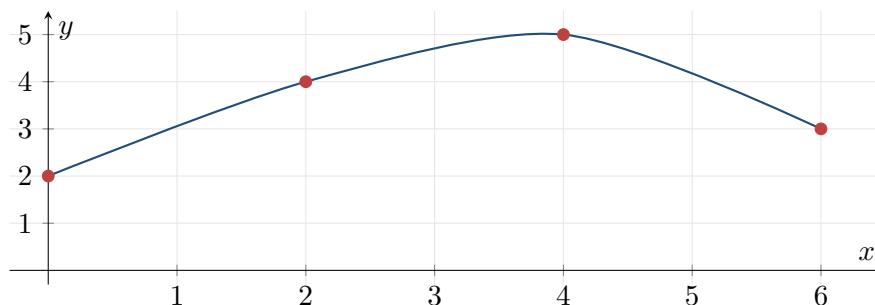
1. Verify that $T_n = \frac{\text{LRAM}_n + \text{RRAM}_n}{2}$ for each row.

2. When n doubles from 4 to 8, by about what factor does the error of T_n shrink?

3. Why is T_n usually much more accurate than either LRAM $_n$ or RRAM $_n$?

10. Trapezoids from a Graph

The graph of f is shown. The marked points have integer coordinates.



Use a trapezoidal sum with three subintervals of width 2 to approximate $\int_0^6 f(x) dx$.

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TRAPEZOIDAL RULE

Mixed Review

11. Mixed Practice

Show your work clearly.

1. Use the Trapezoidal Rule with $n = 4$ to approximate $\int_0^2 (x^2 + 1) dx$.

2. Use the Trapezoidal Rule with $n = 3$ to approximate $\int_1^4 \frac{1}{x^2} dx$. Is your answer too large or too small?

3. A pool is 40 ft long. Its width is measured every 10 ft: 12, 15, 18, 16, and 11 ft. Estimate the area of the pool's surface.

12. Explain Your Thinking

Explain why the Trapezoidal Rule gives the exact value of $\int_a^b f(x) dx$ when f is a linear function.

13. Challenge Problem

Show algebraically that $T_n = \frac{\text{LRAM}_n + \text{RRAM}_n}{2}$ for any n .

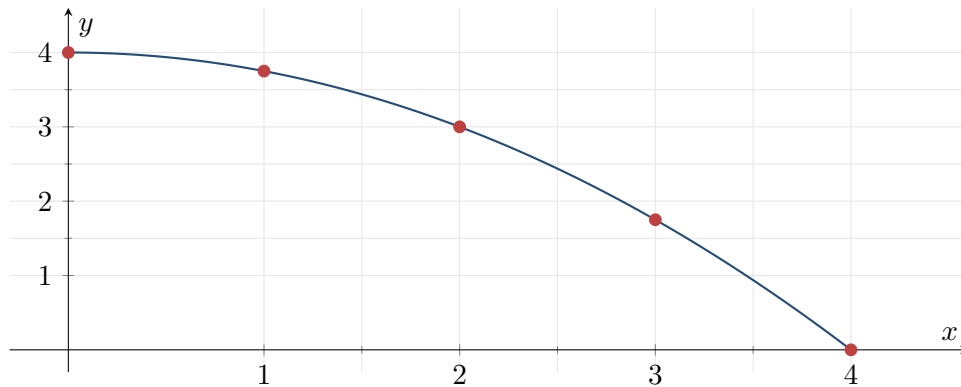
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14. Graphing Challenge

The graph of $g(x) = 4 - \frac{1}{4}x^2$ on $[0, 4]$ is shown.

- Draw the 4 trapezoids for T_4 on the grid.
- Find the trapezoidal estimate of $\int_0^4 g(x) dx$.
- Decide whether your estimate is too high or too low.



1. $T_4 =$ _____

2. Too high or too low? Justify using concavity.

Reflection: What is one thing you understand better about the Trapezoidal Rule after completing this worksheet?