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VELOCITY AND OTHER RATES OF CHANGE

AP Calculus AB Practice Worksheet

Learning Goal: Interpret the derivative as an instantaneous rate of change, analyze motion along a line using position, velocity, speed, and acceleration, model free fall, and apply derivatives to rates in geometry and economics.

1. Motion Along a Line

Position	$s = f(t)$
Displacement	$\Delta s = f(t + \Delta t) - f(t)$
Velocity	$v(t) = \frac{ds}{dt}$ (sign gives direction)
Speed	$ v(t) $ (never negative)
Acceleration	$a(t) = \frac{dv}{dt} = \frac{d^2s}{dt^2}$
Jerk	$j(t) = \frac{da}{dt} = \frac{d^3s}{dt^3}$

2. Instantaneous Rates of Change

1. Find the rate of change of the area of a circle with respect to its radius when $r = 5$ m. Include units.

2. Find the rate of change of the volume of a cube with respect to its edge length s when $s = 3$ in.

3. Find the rate of change of the volume of a sphere with respect to its radius. What familiar formula do you get?

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3. Analyzing Particle Motion

A particle moves along a horizontal line so that its position at time t seconds is

$$s(t) = t^3 - 6t^2 + 9t, \quad 0 \leq t \leq 5,$$

where s is in meters and right is the positive direction.

1. Find the velocity $v(t)$ and the acceleration $a(t)$.

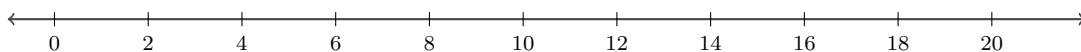
2. When is the particle at rest?

3. Complete the sign chart. Then state when the particle moves right and when it moves left.

Interval	$(0, 1)$	$(1, 2)$	$(2, 3)$	$(3, 5)$
Sign of $v(t)$				
Sign of $a(t)$				
Speeding up or slowing down?				

4. Find the position at $t = 0, 1, 3, 5$. Then find the total distance traveled from $t = 0$ to $t = 5$.

5. Show the particle's path on the number line.



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4. Speeding Up or Slowing Down?

A particle is **speeding up** when $v(t)$ and $a(t)$ have the *same* sign, and **slowing down** when they have *opposite* signs.

1. A particle moves along the x -axis with position

$$x(t) = \frac{t^3}{3} - 4t^2 + 12t, \quad t \geq 0.$$

- (a) Find $v(t)$ and $a(t)$.

- (b) When does the particle change direction?

- (c) Find the speed of the particle at $t = 5$. Is the speed increasing or decreasing at $t = 5$? Justify.

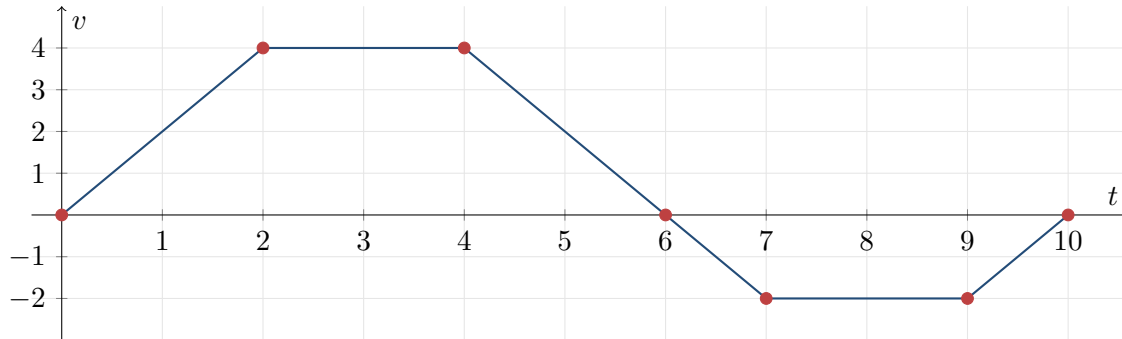
2. For $s(t) = t^2 - 4t + 3$, find every interval on which the particle is speeding up and every interval on which it is slowing down, for $t \geq 0$.

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5. Reading a Velocity Graph

The graph shows the velocity $v(t)$, in ft/sec, of an object moving along a line for $0 \leq t \leq 10$.



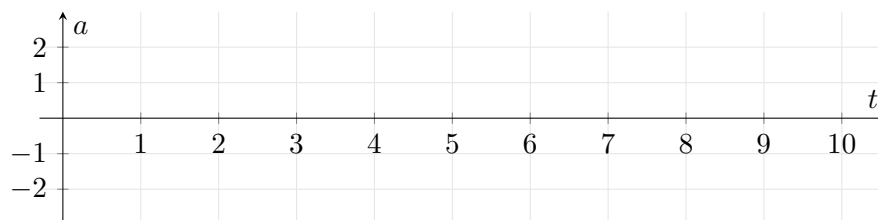
1. When is the object moving forward? Backward? When is it at rest?

2. When is its speed greatest? What is that speed?

3. Find the acceleration at $t = 1$, $t = 3$, $t = 5$, and $t = 8$.

4. On what intervals is the object speeding up? Slowing down?

5. Sketch the graph of the acceleration $a(t)$ on the grid.



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6. Free-Fall Motion

Near Earth's surface, an object moving vertically has height

$$s(t) = s_0 + v_0 t - 16t^2 \text{ (feet)} \quad \text{or} \quad s(t) = s_0 + v_0 t - 4.9t^2 \text{ (meters)}$$

where s_0 is the initial height and v_0 is the initial velocity.

1. A rock is blasted straight up from the ground with an initial velocity of 160 ft/sec, so $s(t) = 160t - 16t^2$.

(a) How high does the rock go?

(b) What are the velocity and speed of the rock when it is 256 ft above the ground on the way up? On the way down?

(c) What is the acceleration at any time t ? _____

(d) When does the rock hit the ground, and how fast is it moving then?

2. A ball is dropped from the top of a 256-ft building. When does it hit the ground, and what is its speed at impact?

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7. Sensitivity to Change

The derivative tells how sensitive one quantity is to small changes in another.

1. A doctor's dosage formula gives the amount of medicine A , in mg, for a patient of mass m kg as $A(m) = 0.02m^2 + 3m$. Find $A'(70)$ and explain its meaning.

2. The time a pendulum takes to swing is $T = 2\pi\sqrt{\frac{L}{9.8}}$ seconds, where L is the length in meters. Find $\frac{dT}{dL}$ when $L = 1$ m. Is T more sensitive to changes in L for short or long pendulums? Explain.

8. Derivatives in Economics

Marginal cost $C'(x)$ approximates the cost of producing *one more* unit: $C(x+1) - C(x) \approx C'(x)$. Marginal revenue $R'(x)$ and marginal profit $P'(x)$ work the same way.

1. The cost of producing x radiators is $C(x) = 8000 + 84x - x^2$ dollars, for $0 \leq x \leq 40$. Find the marginal cost when $x = 10$. Compare it with the exact cost of the 11th radiator, $C(11) - C(10)$.

2. The revenue from selling x phone cases is $R(x) = 20x - 0.01x^2$ dollars. Find the marginal revenue at $x = 500$. What does its value suggest?

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9. AP-Style Problem: Velocity from a Table

A rocket's upward velocity $v(t)$, in meters per second, is differentiable. Selected values are shown.

t (sec)	0	10	20	30	40
$v(t)$ (m/sec)	0	80	140	170	160

1. Use the data to estimate the acceleration of the rocket at $t = 15$ seconds. Include units.

2. Estimate $a(35)$. Is the rocket speeding up or slowing down at $t = 35$? Explain.

3. Is there a time t in $(0, 40)$ at which $a(t) = 0$? Justify your answer.

4. Explain the meaning of $v'(20)$ in the context of the problem.

10. Explain Your Thinking

1. Can an object have a negative velocity and an increasing speed at the same time? Give an example.

2. What is the difference between displacement and total distance traveled?

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Mixed Review

11. Mixed Practice

Show your work clearly.

1. A particle's position is $s(t) = 2t^3 - 15t^2 + 36t$. When is the particle at rest?

2. For the particle in problem 1, find the acceleration when the velocity is 0.

3. On the Moon, a rock thrown upward has height $s(t) = 24t - 0.8t^2$ meters. Find its maximum height.

4. Find the rate of change of the area $A = s^2$ of a square with respect to its side s when $s = 12$ cm.

12. Challenge Problem

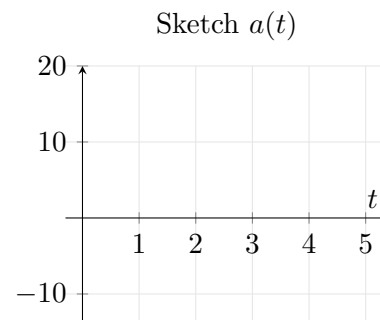
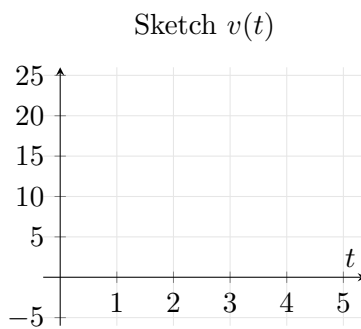
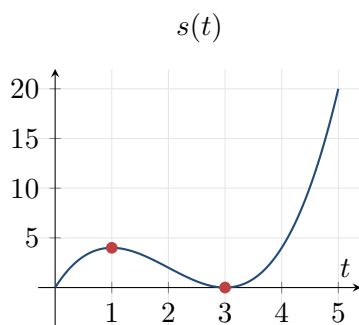
A particle moves along a line with position $s(t) = t^4 - 8t^3 + 18t^2$ for $t \geq 0$. Find every time when the particle is at rest, and determine whether the particle changes direction at each of those times.

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13. Graphing Challenge

The graph of the position function $s(t) = t^3 - 6t^2 + 9t$ from Section 3 is shown. Sketch the velocity and acceleration graphs on the grids below.



1. Where does the graph of $v(t)$ cross the t -axis? What happens to the graph of $s(t)$ at those times?

2. Where does the graph of $a(t)$ cross the t -axis? What does this mean for the graph of $s(t)$?

Reflection: What is one thing you understand better about velocity and other rates of change after completing this worksheet?
