Precalculus

Topic: Double-Angle, Half-Angle, and Product-Sum Formulas

Instructions

Solve the following problems related to double-angle, half-angle, and product-sum formulas. Show all work clearly and check your solutions.

Practice Problems

1. **Use the double-angle formulas to simplify the following expressions:**

   (i) $\sin(2A)$
   (ii) $\cos(2A)$
   (iii) $\tan(2A)$
   (iv) $\sin(2\theta)$
   (v) $\cos(2\theta)$
   (vi) $\tan(2\theta)$
   (vii) $\sin(2A)$ where $A = 45°$
   (viii) $\cos(2A)$ where $A = 30°$
   (ix) $\tan(2A)$ where $A = 15°$
   (x) $\sin(2A + B)$

2. **Prove the following identities using the double-angle formulas:**

   (i) $\cos(2A) = \cos^2 A - \sin^2 A$
   (ii) $\sin(2A) = 2\sin A \cos A$
   (iii) $\tan(2A) = \frac{2\tan A}{1-\tan^2 A}$
   (iv) $\cos(2A) = 2\cos^2 A - 1$
   (v) $\cos(2A) = 1 - 2\sin^2 A$

(vi) $\tan(2A) = \frac{1-\cos(2A)}{2\sin(2A)}$

(vii) Prove $\cos(2A) = \cos^2 A - \sin^2 A$ for $A = 60°$

3. **Use the half-angle formulas to find the exact value of the following expressions:**

(i) $\sin\left(\frac{\pi}{4}\right)$

(ii) $\cos\left(\frac{\pi}{6}\right)$

(iii) $\tan\left(\frac{\pi}{3}\right)$

(iv) $\sin\left(\frac{\pi}{2}\right)$

(v) $\cos\left(\frac{\pi}{3}\right)$

(vi) $\tan\left(\frac{\pi}{4}\right)$

(vii) Find $\sin\left(\frac{\pi}{8}\right)$ using half-angle formula

(viii) $\cos\left(\frac{\pi}{12}\right)$ using half-angle formula

(ix) $\tan\left(\frac{\pi}{6}\right)$ using half-angle formula

(x) $\sin\left(\frac{\theta}{2}\right)$ when $\theta = 30°$

4. **Solve the following equations using the double-angle and half-angle formulas:**

(i) $\sin(2A) = \frac{1}{2}$

(ii) $\cos(2A) = \frac{1}{2}$

(iii) $\tan(2A) = 1$

(iv) $\sin\left(\frac{\theta}{2}\right) = \frac{1}{2}$

(v) $\cos\left(\frac{\theta}{2}\right) = \frac{\sqrt{2}}{2}$

(vi) Solve for A: $\sin(2A) = \sin(60°)$

(vii) Solve for A: $\tan(2A) = \tan(45°)$

(viii) $\cos\left(\frac{\theta}{2}\right) = 0.5$

5. **Use the product-sum formulas to simplify the following trigonometric expressions:**

(i) $\sin A \sin B$

(ii) $\cos A \cos B$

(iii) $\sin A \cos B$

(iv) $\cos A \sin B$

(v) $\sin(A + B)\sin(A - B)$

(vi) $\cos(A + B)\cos(A - B)$

(vii) $\sin(45°)\cos(30°)$

6. **Find the exact value of the following expressions using product-sum formulas:**

(i) $\sin(45°)\sin(30°)$

(ii) $\cos(60°)\cos(45°)$

(iii) $\sin(90°)\cos(45°)$

(iv) $\cos(60°)\sin(30°)$

(v) $\cos(30°)\sin(45°)$

(vi) $\sin(60°)\cos(45°)$

Multiple-Choice Questions

1. What is the formula for $\cos(2A)$?

A. $\cos^2 A - \sin^2 A$

B. $2\cos^2 A - 1$

C. $1 - 2\sin^2 A$

D. All of the above

2. What is the formula for $\tan(2A)$?

A. $\frac{2\tan A}{1-\tan^2 A}$

B. $\frac{\tan A}{1-\tan^2 A}$

C. $\frac{1-\tan A}{2\tan A}$

D. $2\tan A$

3. What is the half-angle formula for $\sin\left(\frac{\theta}{2}\right)$?

A. $\pm\sqrt{\frac{1-\cos\theta}{2}}$

B. $\pm\sqrt{\frac{1+\cos\theta}{2}}$

C. $\frac{1+\cos\theta}{2}$

D. $\frac{1-\cos\theta}{2}$

4. What is the product-sum formula for $\sin A \sin B$?

A. $\frac{1}{2}[\cos(A-B) - \cos(A+B)]$

B. $\frac{1}{2}[\cos(A-B) + \cos(A+B)]$

C. $\frac{1}{2}[\sin(A-B) - \sin(A+B)]$

D. $\frac{1}{2}[\sin(A-B) + \sin(A+B)]$

5. What is the exact value of $\cos(60°)\cos(30°)$ using product-sum formulas?

A. $\frac{1}{2}$

B. $\frac{\sqrt{3}}{4}$

C. $\frac{\sqrt{2}}{2}$

D. $\frac{\sqrt{3}}{2}$