Precalculus

Topic: Plane Curves and Parametric Equations

Instructions

Solve the following problems related to plane curves and parametric equations. Show all work clearly and check your solutions.

Practice Problems

1. **A pair of parametric equations is given. (a) Sketch the curve represented by the parametric equations. (b) Find a rectangular-coordinate equation for the curve by eliminating the parameter.**

   (i) $x = 2t,\ y = t + 6$

   (ii) $x = 6t - 4,\ y = 3t,\ t \geq 0$

   (iii) $x = t^2,\ y = t - 2,\ 2 \leq t \leq 4$

   (iv) $x = 2t + 1,\ y = (t + \frac{1}{2})^2$

   (v) $x = \frac{1}{t^2},\ y = t + 1$

   (vi) $x = t + 1,\ y = \frac{t}{t+1}$

   (vii) $x = 4t^2,\ y = 8t^3$

   (viii) $x = |t|,\ y = 1 - |t|$

2. **The position of an object in circular motion is modeled by the given parametric equations. Describe the path of the object by stating the radius of the circle, the position at time $t = 0$, the orientation of the motion (clockwise or counterclockwise), and the time it takes to complete one revolution around the circle.**

   (i) $x = 3\cos t,\ y = 3\sin t$

   (ii) $x = 2\sin t,\ y = 2\cos t$

   (iii) $x = \sin 2t,\ y = \cos 2t$

   (iv) $x = 4\cos 3t,\ y = 4\sin 3t$

3. **Find parametric equations for the line with the given properties.**

(i) Slope $\frac{1}{2}$, passing through $(4, -1)$

(ii) Slope -2, passing through $(-10, -20)$

(iii) Passing through $(6, 7)$ and $(7, 8)$

(iv) Passing through $(12, 7)$ and the origin

(v) Find parametric equations for the circle $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

(vi) Find parametric equations for the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

4. **Show by eliminating the parameter t that the following parametric equations represent a hyperbola:**

(i) $x = a \tan t,\ y = b \sec t$ (ii) $x = a \cos t,\ y = b\sqrt{1 + t}$

5. **Sketch the curve given by the parametric equations.**

(i) $x = \cos t,\ y = \sin t,\ t \geq 0$ (iii) $x = 3t,\ y = 3t^2$

(ii) $x = \sin t,\ y = \sin 2t$ (iv) $x = \frac{3t}{1+t^3},\ y = \frac{3t^2}{1+t^3}$

6. **Find parametric equations for the following curves:**

(i) $x = \cos\theta,\ y = 2\sin\theta,\ 0 < \theta < \pi$ (ii) $x = 2\cos\theta + 2$

7. **A projectile is fired with an initial speed of v_0 ft/s at an angle α above the horizontal. Its position after t seconds is given by the parametric equations:**

$$x = v_0 \cos\alpha \cdot t, \quad y = v_0 \sin\alpha \cdot t - 16t^2$$

(where x and y are measured in feet). Show that the path of the projectile is a parabola by eliminating the parameter t.