Precalculus

Topic: Polar Form of Complex Numbers; De Moivre's Theorem

Instructions

Solve the following problems related to the polar form of complex numbers and De Moivre's Theorem. Show all work clearly and check your solutions.

Practice Problems

1. **Graph the Complex Number and Find its Modulus.**

   (i) $4i$

   (ii) -2

   (iii) $5+2i$

   (iv) $\sqrt{3}+i$

   (v) $3+4i$

   (vi) $-1-\frac{\sqrt{3}}{3}i$

   (vii) 6

   (viii) $7-3i$

2. **Sketch the Complex Number z, and Also Sketch $2z, -z$, and $\frac{z}{2}$ on the Same Complex Plane.**

   (i) $z=2+i$

   (ii) $z=-1+i$

   (iii) $z=8+2i$

   (iv) $z=-1-i$

3. **Sketch the Complex Number z and its Complex Conjugate $\overline{z}$ on the Same Complex Plane.**

(i) $z = 2 + i$

(ii) $z = 5 - 6i$

(iii) $z = -1 + i$

(iv) $z = -5 + 6i$

4. **Sketch the Set in the Complex Plane.**

(i) $z = a + bi \quad (a \leq 0, b \geq 0)$

(ii) $z = a + bi \quad (a \geq 0, b \leq 0)$

(iii) $z = a + bi \quad (|z| \leq 3)$

(iv) $z = a + bi \quad (|z| < 2)$

(v) $z = a + bi \quad (|z| = 1)$

(vi) $z = a + bi \quad (|z| \geq 1)$

(vii) $z = a + bi \quad (|z| \leq 2, a \geq 0)$

(viii) $z = a + bi \quad (a \geq b)$

5. **Write the Complex Number in Polar Form with Argument θ Between 0 and 2π.**

(i) $1 + i$

(ii) $-1 + i$

(iii) $\sqrt{3} - i$

(iv) $2 - 3i$

(v) $-2 - 2i$

(vi) $3 + 3i$

(vii) $5 - 4i$

(viii) $2 + i$

6. **Find the Product $z_1 z_2$ and the Quotient $\frac{z_1}{z_2}$. Express Your Answer in Polar Form.**

(i) $z_1 = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4}, \quad z_2 = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$

(ii) $z_1 = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4}, \quad z_2 = 5\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

(iii) $z_1 = 5\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right), \quad z_2 = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

(iv) $z_1 = 4\left(\cos 120^\circ + i\sin 120^\circ\right), \quad z_2 = 2\left(\cos 30^\circ + i\sin 30^\circ\right)$

(v) $z_1 = \sqrt{2}\left(\cos 75^\circ + i\sin 75^\circ\right), \quad z_2 = 3\left(\cos 60^\circ + i\sin 60^\circ\right)$

(vi) $z_1 = 25\left(\cos 150^\circ + i\sin 150^\circ\right), \quad z_2 = 3\left(\cos 155^\circ + i\sin 155^\circ\right)$

7. **Write z_1 and z_2 in Polar Form, and then find the Product $z_1 z_2$ and the Quotients $\frac{z_1}{z_2}$ and $\frac{1}{z_1}$.**

(i) $z_1 = \sqrt{3} + i, \quad z_2 = 2 + i$

(ii) $z_1 = 4 + 4i, \quad z_2 = -1 + 2i$

(iii) $z_1 = 3 - 2i, \quad z_2 = -3 + 3i$

(iv) $z_1 = 1 + 3i, \quad z_2 = 2 - 3i$

(v) $z_1 = 5 + 5i, \quad z_2 = 3 - 2i$

(vi) $z_1 = -1 + i, \quad z_2 = -1 - i$

8. **Find the Indicated Power Using De Moivre's Theorem.**

(i) $(1 + i)^{20}$

(ii) $(2\sqrt{3} + 2i)^5$

(iii) $(1 - i)^8$

(iv) $(2 - 2i)^{12}$

(v) $(1 + i)^6$

(vi) $(1 - i)^4$

(vii) $(2 + i)^5$

(viii) $(1 + \sqrt{3}i)^3$

Multiple Chioce Questions

(1) Which of the following represents the polar form of the complex number $z = -1 + i$?

A. $r = \sqrt{2}, \theta = \frac{3\pi}{4}$

B. $r = 1, \theta = \frac{\pi}{4}$

C. $r = 1, \theta = \frac{5\pi}{4}$

D. $r = 2, \theta = \frac{\pi}{2}$

(2) What is the result of $(1 + i)^6$ using De Moivre's Theorem?

A. $-2 + 2i$

B. 8

C. 2

D. $-2 - 2i$

(3) For the complex number $z = 3 + 4i$, the modulus of z is:

A. 5

B. 7

C. 1

D. 25

(4) What is the polar form of the complex number $z = 2 - 2i$?

A. $r = 2, \theta = \frac{\pi}{4}$

B. $r = 2\sqrt{2}, \theta = \frac{5\pi}{4}$

C. $r = 2\sqrt{2}, \theta = \frac{\pi}{4}$

D. $r = 4, \theta = \frac{5\pi}{4}$