Precalculus

Topic: Three-Dimensional Coordinate Geometry

Instructions

Solve the following problems related to three-dimensional coordinate geometry. Show all work clearly and check your solutions.

Practice Problems

1. **Two points P and Q are given. (a) Plot P and Q. (b) Find the distance between P and Q.**

   (i) $P(3, 1, 0), Q(-1, 2, -5)$

   (ii) $P(5, 0, 10), Q(3, -6, 7)$

   (iii) $P(-2, -1, 0), Q(-12, 3, 0)$

   (iv) $P(5, -4, -6), Q(8, -7, 4)$

2. **Describe and sketch the surface represented by the given equation.**

   (i) $x = 4$

   (ii) $y = -2$

   (iii) $z = 8$

   (iv) $y = -1$

3. **Find an equation of a sphere with the given radius r and center C.**

   (i) $r = 5; C(2, -5, 3)$

   (ii) $r = 3; C(-1, 4, -7)$

   (iii) $r = \sqrt{6}; C(3, 1, 0)$

   (iv) $r = \sqrt{11}; C(-10, 0, 1)$

4. **Show that the equation represents a sphere, and find its center and radius.**

(i) $x^2 + y^2 + z^2 - 10x + 2y + 8z = 9$ (iii) $x^2 + y^2 + z^2 = 12x + 2y$

(ii) $x^2 + y^2 + z^2 + 4x - 6y + 2z = 10$ (iv) $x^2 + y^2 + z^2 = 14y - 6z$

5. **Describe the trace of the sphere** $(x + 1)^2 + (y - 2)^2 + (z + 10)^2 = 100$ **in:**

   (i) the xy-plane (ii) the plane $x = 4$

6. **Describe the trace of the sphere** $x^2 + (y - 4)^2 + (z - 3)^2 = 144$ **in:**

   (i) the xz-plane (ii) the plane $z = -2$

7. **Application Problems**

   (i) A drone is flying at a height of 100 meters above the ground at position $P(50, 30, 100)$. If the drone moves to a position $Q(100, 80, 100)$, find the distance it traveled.

   (ii) A ball is thrown from point $A(0, 0, 0)$ with initial velocity $\mathbf{v} = \langle 5, 3, 2\rangle$. Find the position of the ball after 3 seconds if the motion is modeled by the equation $\mathbf{r}(t) = \langle 5t, 3t, 2t\rangle$.

Multiple Chioce Questions

(1) What is the center of the sphere with equation $x^2 + y^2 + z^2 - 4x + 6y + 2z = 10$?

   (a) $(2, -3, -1)$
   (b) $(-2, 3, 1)$
   (c) $(4, -6, -2)$
   (d) $(-4, 6, 2)$

(2) What is the radius of the sphere represented by the equation $x^2+y^2+z^2-6x-4y-8z = -10$?

   (a) 4
   (b) 3
   (c) 5
   (d) 2

(3) What is the trace of the sphere $x^2 + y^2 + z^2 = 9$ in the yz-plane?

   (a) A circle with radius 3
   (b) A point at the origin
   (c) A line
   (d) A circle with radius 9