Precalculus

Topic: Trigonometric Identities

Instructions

Solve the following problems related to trigonometric identities. Show all work clearly and check your solutions.

Practice Problems

1. **Simplify the following trigonometric expressions:**

   (i) $\sin^2\theta + \cos^2\theta$

   (ii) $1 + \tan^2\theta$

   (iii) $1 - \sin^2\theta$

   (iv) $\sec^2\theta - \tan^2\theta$

   (v) $\csc^2\theta - \cot^2\theta$

   (vi) $\sin^2\theta + \cos^2\theta - 1$

   (vii) $\tan^2\theta + 1 = \sec^2\theta$

   (viii) $\sec\theta - \cos\theta$

2. **Prove the following identities:**

   (i) $\sin^2\theta + \cos^2\theta = 1$

   (ii) $1 + \tan^2\theta = \sec^2\theta$

   (iii) $\csc^2\theta - \cot^2\theta = 1$

   (iv) $1 + \cot^2\theta = \csc^2\theta$

   (v) $\sin(2\theta) = 2\sin\theta\cos\theta$

   (vi) $\cos(2\theta) = \cos^2\theta - \sin^2\theta$

   (vii) $\sin(A + B) = \sin A\cos B + \cos A\sin B$

3. **Verify the identity.**

(i) $\frac{\sin\theta}{\tan\theta} = \cos\theta$

(ii) $\tan x = \frac{\sin x}{\sec x}$

(iii) $\cos u \sec u = \cot u$

(iv) $\sin B + \cos B \cot B = \csc B$

(v) $\cos(-x) - \sin(-x) = \cos x + \sin x$

(vi) $\cot(-x)\cos(-a) + \sin(-a) = -\csc x$

(vii) $\csc x[\csc x - \sin(-x)] = \cot^2 x$

(viii) $\tan\theta + \cot\theta = \sec\theta \csc\theta$

(ix) $(\sin x + \cos x)^2 = 1 + 2\sin x \cos x$

(x) $(1 - \cos\beta)(1 + \cos\beta) = \frac{1}{\csc^2\beta}$

(xi) $\cos x + \sec x = \frac{\sin x}{\sec x}$

(xii) $(\sin x + \cos x)^2 = \frac{\sin^2 x - \cos^2 x}{\sin x - \cos x}$

(xiii) $\sin^2 x + \cos^2 x = 1$

4. **Find the value of the following trigonometric expressions:**

(i) $\sin 30° + \cos 60°$

(ii) $\tan 45°$

(iii) $\sin 90°$

(iv) $\cos 0°$

(v) $\cot 90°$

(vi) $\sin 45°$

(vii) $\cos 120°$

5. **Solve the following trigonometric equations:**

(i) $\sin\theta = \frac{1}{2}$

(ii) $\cos\theta = \frac{\sqrt{3}}{2}$

(iii) $\tan\theta = 1$

(iv) $\sec\theta = 2$

(v) $\cot\theta = \frac{\sqrt{3}}{3}$

Multiple-Choice Questions

1. What is the value of $\sin^2\theta + \cos^2\theta$?

A. 1

B. 2

C. 0

D. $\sin^2\theta$

2. What is the identity for $\cos(2\theta)$?

A. $\cos^2\theta - \sin^2\theta$

B. $2\cos^2\theta - 1$

C. $1 - 2\sin^2\theta$

D. All of the above

3. What is the value of $\sec^2\theta - \tan^2\theta$?

A. 1

B. $\sin^2\theta$

C. $\cos^2\theta$

D. None of the above