Precalculus

Topic: Dividing Polynomials

Instructions

Solve the following problems related to dividing polynomials. Show all work clearly and check your solutions.

Practice Problems

1. **Find the quotient and remainder using long division**

   (i) $\frac{x^3-3x^2+2x-5}{x-2}$

   (ii) $\frac{2x^4+3x^3-5x^2+4}{x+1}$

   (iii) $\frac{x^3-4x^2+5x-6}{x-3}$

   (iv) $\frac{3x^4-5x^3+x^2-2}{x-1}$

   (v) $\frac{x^3-5x^2+2x-3}{x+2}$

2. **Find the quotient and remainder using synthetic division.**

   (i) $\frac{x^3+2x^2-3x+4}{x+1}$

   (ii) $\frac{3x^4-5x^3+2x-7}{x-2}$

   (iii) $\frac{x^3-2x^2+5x-8}{x+3}$

   (iv) $\frac{x^4-4x^3+3x-6}{x-1}$

3. **Use the Factor Theorem to show that $x-c$ is a factor of $P(x)$ for the given values of c.**

(i) $P(x) = x^3 - 3x^2 + 3x - 1, \quad c = 1$

(ii) $P(x) = x^3 + 2x^2 - 3x - 10, \quad c = 2$

(iii) $P(x) = 2x^3 + 7x^2 + 6x - 5, \quad c = \frac{1}{2}$

(iv) $P(x) = x^4 + 3x^3 - 16x^2 - 27x + 63, \quad c = 3$

4. **Show that the given value(s) of c are zeros of $P(x)$, and find all other zeros of $P(x)$.**

   (i) $P(x) = x^3 - x^2 - 11x + 15, \quad c = 3$

   (ii) $P(x) = 3x^4 - x^3 - 21x^2 - 11x + 6, \quad c = \frac{1}{3}$

5. **Find a polynomial of the specified degree that has the given zeros.**

   (i) Degree 3: zeros $-1, 1, 3$

   (ii) Degree 4: zeros $-2, 0, 2, 4$

   (iii) Degree 4: zeros $-1, 3, 5, -2$

   (iv) Degree 5: zeros $-2, -1, 0, 1, 2$

6. **Solve the following word problems involving polynomial division.**

   (i) A polynomial $P(x) = 4x^3 - 5x^2 + 6x - 7$ represents the total cost to produce x units of a product. The fixed cost per unit is given by the polynomial $Q(x) = 2x - 1$. Divide $P(x)$ by $Q(x)$ to find the cost per unit produced.

   (ii) A company manufactures a product whose total revenue is given by the polynomial $R(x) = 3x^4 - 2x^3 + 4x - 9$, where x is the number of units sold. If the price per unit is given by $P(x) = x^2 - 3x + 2$, divide $R(x)$ by $P(x)$ to determine the number of units sold.

Multiple-Choice Questions

1. What is the quotient when dividing $x^4 - 4x^3 + 3x - 6$ by $x - 1$?

   A. $x^3 - 3x^2 + 3x - 6$

   B. $x^3 - 4x^2 + 3$

   C. $x^3 - 4x^2 + 3x - 6$

   D. $x^3 - 4x^2 + 4x - 6$

2. What is the quotient when dividing $3x^3 - 6x^2 + 9x - 12$ by $3x - 3$?

   A. $x^2 - 2x + 3$

   B. $x^2 - 2x + 4$

   C. $x^2 - 2x + 1$

   D. $x^2 - 3x + 2$

3. When dividing $2x^3 - x^2 + 3x - 5$ by $x - 2$, the remainder is:

   A. 1

B. -1

C. 0

D. 2

4. What is the result of dividing $4x^4 - 2x^3 + 5x - 3$ by $2x^2 + x - 1$?

   A. $2x^2 - x + 3$

   B. $2x^2 + x - 2$

   C. $2x^2 + x - 3$

   D. $2x^2 + 2x - 3$

5. What is the quotient when dividing $x^4 + 5x^3 - 2x^2 + 4x - 7$ by $x + 1$?

   A. $x^3 + 4x^2 - 6x + 3$

   B. $x^3 + 5x^2 - 7x + 2$

   C. $x^3 + 4x^2 - 6x + 5$

   D. $x^3 + 4x^2 - 5x + 3$