Precalculus

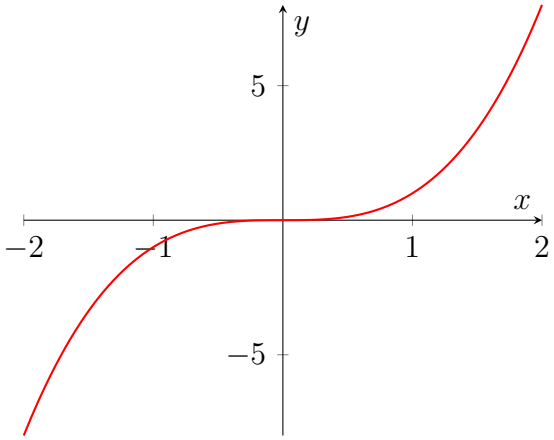
Topic: One-to-One Functions and Their Inverses

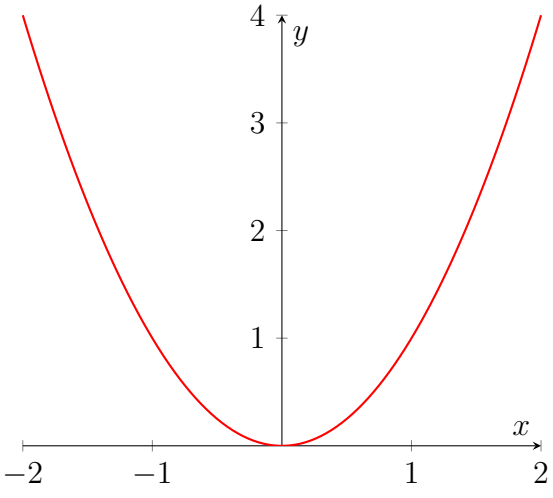
Instructions

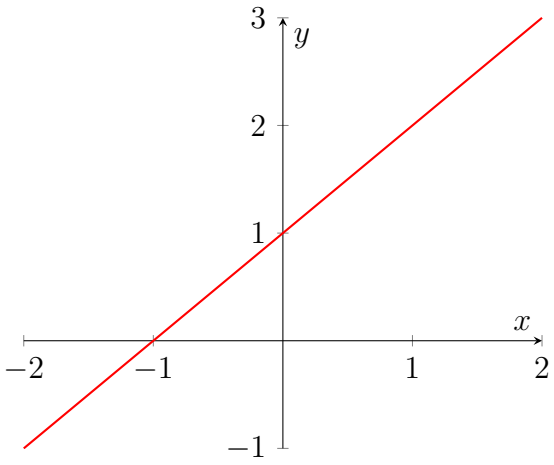
Solve the following problems related to one-to-one functions and their inverses. Show all work clearly and check your solutions.

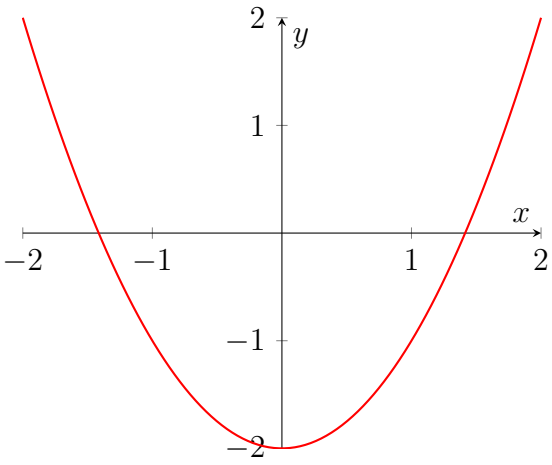
Practice Problems

1. **The graph of a function f is given. Determine whether f is one-to-one.**









2. **Determine whether the following functions are one-to-one.**

   (i) $f(x) = 2x + 3$

   (ii) $g(x) = x^2 + 4$

   (iii) $h(x) = 3x - 1$

   (iv) $j(x) = \frac{1}{x}$

   (v) $k(x) = x^3$

3. **Find the inverse of the following functions (if they exist).**

   (i) $f(x) = 2x + 5$

   (ii) $g(x) = \frac{3x+2}{x-1}$

   (iii) $h(x) = x^2 + 4x$ (Check if it is one-to-one before finding the inverse)

   (iv) $j(x) = \sqrt{x+1}$

   (v) $k(x) = 3x - 4$

4. **Find the inverse of the following functions by solving for y and then expressing y as a function of x.**

   (i) $f(x) = 3x + 2$

   (ii) $g(x) = 5x - 1$

   (iii) $h(x) = \frac{x+1}{x-2}$

   (iv) $j(x) = \frac{1}{x+2}$

   (v) $k(x) = x^3 + 3$

5. **Verify whether the following functions are inverses of each other by checking if $f(g(x)) = x$ and $g(f(x)) = x$.**

   (i) $f(x) = 2x + 1$ and $g(x) = \frac{x-1}{2}$

   (ii) $f(x) = x^2 + 3$ and $g(x) = \sqrt{x-3}$

   (iii) $f(x) = \frac{1}{x+1}$ and $g(x) = \frac{1}{x} - 1$

   (iv) $f(x) = x^3$ and $g(x) = \sqrt[3]{x}$

(v) $f(x) = 4x + 7$ and $g(x) = \frac{x-7}{4}$

6. **Graph the following functions and identify their inverses visually.**

   (i) $f(x) = x + 2$

   (ii) $g(x) = x^2$ (For domain $x \geq 0$)

   (iii) $h(x) = \sqrt{x}$

   (iv) $j(x) = x^3$

   (v) $k(x) = 2x - 5$

Multiple-Choice Questions

1. Which of the following is true for a one-to-one function?

   A. A one-to-one function passes the horizontal line test.

   B. A one-to-one function has multiple outputs for one input.

   C. A one-to-one function does not have an inverse.

   D. A one-to-one function is always a linear function.

2. Which of the following is the inverse of $f(x) = \frac{3x+1}{x+2}$?

   A. $f^{-1}(x) = \frac{x+2}{3x-1}$

   B. $f^{-1}(x) = \frac{2x-1}{3x+1}$

   C. $f^{-1}(x) = \frac{x-2}{3x+1}$

   D. $f^{-1}(x) = \frac{3x-1}{x+2}$

3. Which of the following is not a one-to-one function?

   A. $f(x) = x^3$

   B. $g(x) = x^2$

   C. $h(x) = 2x + 1$

   D. $j(x) = \frac{1}{x}$

4. What is the inverse of $f(x) = 5x - 4$?

   A. $f^{-1}(x) = \frac{x+4}{5}$

   B. $f^{-1}(x) = \frac{x-4}{5}$

   C. $f^{-1}(x) = 5x + 4$

   D. $f^{-1}(x) = \frac{5}{x-4}$