Precalculus

Topic: The Natural Exponential Function

Instructions

Solve the following problems. Show all work clearly and check your solutions.

Practice Problems

1. **Complete the table of values, rounded to two decimal places, and sketch a graph of the function.**

   (i) $f(x) = 3e^x$

x	$f(x)$
-2	
-1	
-0.5	
0	
0.5	
1	
2	

   (ii) $f(x) = 2e^{-0.5x}$

x	$f(x)$
-3	
-2	
-1	
0	
1	
2	
3	

2. **Graph the function. State the domain, range, and asymptote.**

   (i) $f(x) = -e^x$

   (ii) $y = e^{-x} - 1$

   (iii) $f(x) = e^{x^2}$

   (iv) $h(x) = e^{x+1} - 3$

(v) The hyperbolic cosine function is defined by

$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

- Sketch the graphs of the functions $y = \frac{1}{2}e^x$ and $y = \frac{1}{2}e^{-x}$ on the same axes, and use graphical addition to sketch the graph of $y = \cosh(x)$.
- Use the definition to show that $\cosh(-x) = \cosh(x)$.

3. **Graph the following functions and identify key characteristics (domain, range, asymptote, intercepts):**

(i) $f(x) = e^x$
(ii) $f(x) = e^{-x}$
(iii) $f(x) = 2e^x$
(iv) $f(x) = e^{x+2}$
(v) $f(x) = 3e^{-x}$

4. **Sketch the graph of $y = e^x$ and its transformation to $y = e^{x+3} - 2$. Discuss how the graph shifts.**

5. **Solve the following application problems:**

(i) The population of a city grows according to the function $P(t) = 5000e^{0.03t}$, where t is the time in years. Find the population after 10 years.
(ii) The amount of money in an investment account grows according to the formula $A(t) = 1500e^{0.05t}$. How much money will be in the account after 8 years?

Multiple-Choice Questions

1. What is the value of e^x when $x = 0$?

A. 0
B. 1
C. e
D. ∞

2. Which of the following is the graph of $y = e^x$?

A. A decreasing curve approaching 0.
B. An increasing curve with horizontal asymptote at $y = 0$.
C. A horizontal line at $y = 0$.
D. A vertical line at $x = 0$.

3. What is the domain of the function $f(x) = e^x$?

A. $x \in (-\infty, \infty)$
B. $x \in [0, \infty)$
C. $x \in (-\infty, 0)$
D. $x \in (-\infty, \infty)$ and $y \in (0, \infty)$