Precalculus

Topic: Transformations of Functions

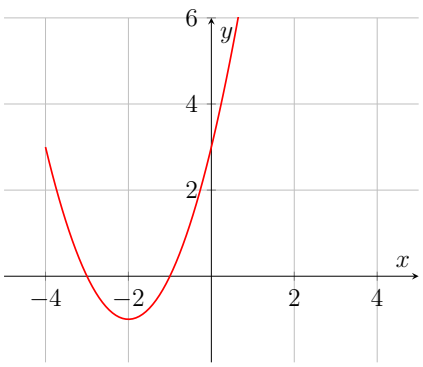
Instructions

Answer the following problems.

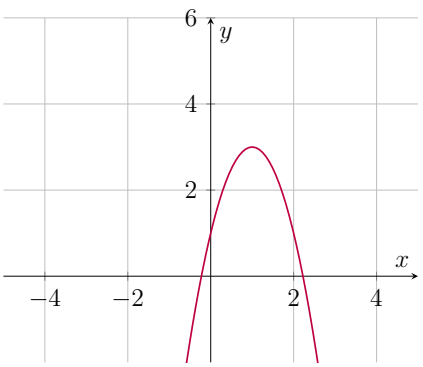
Practice Problems

1. For $f(x) = x^2$, describe and sketch the graph of $f(x-3)+2$. Identify all transformations.
2. Given $f(x) = |x|$, describe the transformations for $-f(x+1)-4$.
3. For $f(x) = \sin(x)$, determine the transformations and sketch the graph of $2f(x - \frac{\pi}{2}) + 1$.
4. If $f(x) = \sqrt{x}$, describe and sketch the transformations for $\frac{1}{2}f(2x) - 3$.

   Describe the transformations for $f(x-2)+3$ and sketch the graph.
5. Identify the transformations applied to the base function in the graphs below.



a.



b.

6. A function f is given, and the indicated transformations are applied to its graph in the given order. Write the equation for the final transformed graph:

   (a) $f(x) = x^2$: Shift 4 units to the right and 3 units upward.

   (b) $f(x) = \sqrt{x}$: Reflect in the x-axis and shift 2 units downward.

   (c) $f(x) = |x|$: Shrink vertically by a factor of $\frac{1}{2}$ and shift 5 units to the left.

   (d) $f(x) = x^3$: Stretch vertically by a factor of 3, reflect in the y-axis, and shift 1 unit upward.

7. Find the formula for $g(x)$ in terms of $f(x)$ given the following transformations of the base graph $f(x)$:

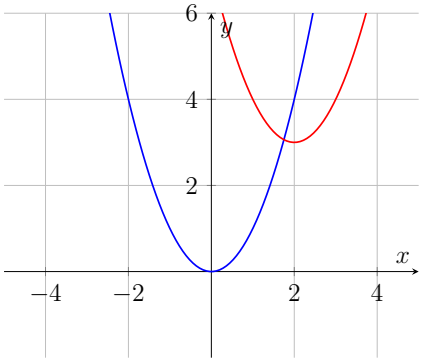
   (a) $g(x)$ is obtained by shifting $f(x)$ 2 units to the left and reflecting it in the x-axis.

   (b) $g(x)$ is obtained by stretching $f(x)$ vertically by a factor of 2 and shifting it 3 units downward.

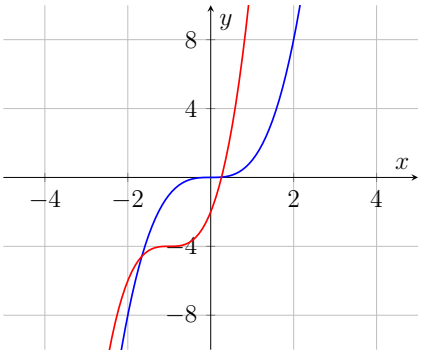
   (c) $g(x)$ is obtained by compressing $f(x)$ horizontally by a factor of $\frac{1}{3}$ and shifting it 4 units to the right.

   (d) $g(x)$ is obtained by reflecting $f(x)$ in the y-axis and shifting it 5 units upward.

8. The graphs of $f(x)$ and $g(x)$ are given. Find a formula for the function $g(x)$ based on the transformations of $f(x)$.



a.



b.

Multiple Choice Questions

1. Which transformation is represented by $f(x-3)+2$?

   a. Shift 3 units left, 2 units up

   b. Shift 3 units right, 2 units up

   c. Shift 3 units left, 2 units down

   d. Shift 3 units right, 2 units down

2. For $f(x) = |x|$, what does $-f(x+2) - 4$ represent?

   a. Reflection over x-axis, shift 2 units left, 4 units down

b. Reflection over x-axis, shift 2 units right, 4 units down

c. Reflection over y-axis, shift 2 units left, 4 units down

d. Reflection over y-axis, shift 2 units right, 4 units up

3. The function $f(x) = \sin(x)$ is transformed to $2f(x - \pi) + 1$. What are the transformations?

   a. Horizontal shift π units left, vertical stretch by 2, shift 1 unit up

   b. Horizontal shift π units right, vertical stretch by 2, shift 1 unit up

   c. Horizontal shift π units right, vertical compression by 2, shift 1 unit down

   d. Horizontal shift π units left, vertical stretch by 2, shift 1 unit down

4. What is the result of applying $f(\frac{x}{2}) - 4$ to $f(x) = x^2$?

   a. Horizontal stretch by 2, shift 4 units down

   b. Horizontal compression by 2, shift 4 units down

   c. Horizontal stretch by 2, shift 4 units up

   d. Horizontal compression by 2, shift 4 units up
