O Level Maths

Topic: Determinants and Inverses

Instructions

Answer all questions. Show all necessary steps. Use determinant and inverse formulas appropriately.

Practice Problems

Determinants of 2x2 and 3x3 Matrices:

1. Find the determinant of:
$$A = \begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$$
2. Find the determinant of:
$$B = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$$
3. Evaluate the determinant of:
$$C = \begin{bmatrix} -1 & 2 \\ 5 & -3 \end{bmatrix}$$
4. Find the value of x such that the matrix
$$D = \begin{bmatrix} x & 1 \\ 2 & x \end{bmatrix}$$
is singular.

Inverses of 2x2 Matrices:

5. Find the inverse of the matrix:
$$A = \begin{bmatrix} 3 & 4 \\ 2 & 1 \end{bmatrix}$$

6. Determine if the matrix
$$B = \begin{bmatrix} 5 & -2 \\ -4 & 1 \end{bmatrix}$$
is invertible. If so, find the inverse.

7. Let
$$M = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$
and verify that $M \cdot M^{-1} = I$, where I is the identity matrix.

8. Find the inverse of
$$P = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$$
and hence solve the system:
$$P \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

Multiple-Choice Questions

1. The determinant of $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ is:

   A. -2

   B. 10

   C. -5

   D. -6

2. Which matrix is non-invertible?

   A. $\begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$

   B. $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$

   C. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

   D. $\begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$

3. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then A^{-1} is:

   A. $\frac{1}{ad-bc} \begin{bmatrix} a & -b \\ -c & d \end{bmatrix}$

   B. $\frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

C. $\frac{1}{ad+bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

D. $\frac{1}{ad-bc}\begin{bmatrix} -d & b \\ c & -a \end{bmatrix}$

4. What is the value of the determinant of an identity matrix?

   A. 0

   B. 1

   C. -1

   D. It depends on the size

5. The determinant of a 3x3 matrix is zero. What can be said about the matrix?

   A. It has an inverse

   B. It is non-singular

   C. It is singular and not invertible

   D. It is diagonal