A Level Maths

Topic: Integration by Substitution and by Parts

Instructions

Answer all questions. Show complete working. Unless otherwise stated, give exact answers.

Practice Questions

Integration by Substitution

1. Evaluate $\int 2x\sqrt{1+x^2}\,dx$ using substitution.
2. Find $\int \dfrac{6x}{(x^2+3)^2}\,dx$
3. Use the substitution $u = \tan x$ to evaluate $\int \sec^2 x \ln(\tan x)\,dx$
4. Evaluate $\displaystyle\int_0^1 \frac{x}{\sqrt{1+x^2}}\,dx$

Integration by Parts

5. Evaluate $\int xe^x\,dx$
6. Find $\int x\ln x\,dx$
7. Calculate $\int x\cos x\,dx$
8. Evaluate $\displaystyle\int_0^{\ln 2} xe^x\,dx$

Multiple-Choice Questions

1. $\int x \ln x \, dx =$

   A. $\frac{x^2}{2} \ln x - \frac{x^2}{4} + C$
   B. $\frac{x^2}{2} \ln x - \frac{x^2}{2} + C$
   C. $x \ln x + C$
   D. $\ln x + x^2 + C$

2. Integration by parts formula is:

   A. $\int u'v \, dx = uv - \int uv' \, dx$
   B. $\int uv' \, dx = uv - \int vu' \, dx$
   C. $\int uv \, dx = uv - \int vu \, dx$
   D. $\int u \, dx = uv + \int v \, dx$

3. Let $I = \int x \cos x \, dx$. Then $I =$

   A. $x \sin x + \cos x + C$
   B. $x \cos x - \int \sin x \, dx$
   C. $x \sin x + \cos x + C$
   D. $x \sin x + \int \cos x \, dx$

4. For substitution, if $u = x^2 + 1$, then $dx =$

   A. $\frac{du}{2x}$
   B. $\frac{dx}{du}$
   C. $2x \, du$
   D. $\frac{1}{2x} \, dx$

5. $\int x^2 e^x \, dx =$

   A. $x^2e^x - 2xe^x + 2e^x + C$
   B. $x^2e^x + 2xe^x + 2e^x + C$
   C. $x^2e^x - xe^x + e^x + C$
   D. $e^x(x^2 + x + 1) + C$