A Level Maths

Topic: Recurrence Relations

Instructions

Answer all questions. Show all necessary steps. Use the recurrence relation to find the required terms or closed-form expressions.

Practice Problems

Generating Terms from Recurrence Relations:

1. Given $u_1 = 3$ and $u_{n+1} = u_n + 4$, find u_2, u_3, u_4, u_5.
2. The recurrence relation is $u_1 = 2$, $u_{n+1} = 2u_n + 1$. Find the first 5 terms.
3. For $u_1 = 5$ and $u_{n+1} = 3u_n - 2$, find u_2 to u_5.
4. The recurrence relation is $u_1 = 1$, $u_{n+1} = \frac{1}{2}u_n$. Find u_2, u_3, u_4.

Finding Closed Form and Solving Problems:

5. The recurrence relation is $u_1 = 3$, $u_{n+1} = u_n + 5$. Find a formula for u_n in terms of n.
6. Given $u_1 = 2$, $u_{n+1} = 3u_n$, show that $u_n = 2 \cdot 3^{n-1}$.
7. For the recurrence $u_1 = 7$, $u_{n+1} = u_n - 2$, find the smallest n such that $u_n \leq -1$.
8. The recurrence relation is $u_1 = 4$, $u_{n+1} = 2u_n + 3$. Find u_2, u_3, and then write an expression for u_4.

Multiple-Choice Questions

1. Given $u_1 = 5$, $u_{n+1} = u_n + 3$. What is u_4?

   A. 11

   B. 14

   C. 17

   D. 18

2. If $u_1 = 2$ and $u_{n+1} = 2u_n$, what is u_5?

   A. 16

   B. 24

   C. 32

   D. 48

3. The recurrence $u_1 = 1$, $u_{n+1} = 3u_n + 1$. Find u_3.

   A. 9

   B. 10

   C. 13

   D. 14

4. Which of the following is a correct closed-form formula for u_n if $u_1 = 2$ and $u_{n+1} = 2u_n$?

   A. $u_n = 2n$

   B. $u_n = 2^n$

   C. $u_n = 2 \cdot 2^{n-1}$

   D. $u_n = 2 + 2n$

5. Given $u_1 = 10$, $u_{n+1} = u_n - 3$, for what value of n is $u_n = 1$?

   A. 2

   B. 3

   C. 4

   D. 4