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LIMITS

AP Calculus AB Practice Worksheet

Learning Goal: Estimate limits from tables and graphs, evaluate limits using limit properties and algebra, find one-sided limits, apply the Sandwich Theorem, and recognize when a limit does not exist.

1. Estimate Limits from Tables

1. Complete the table for $f(x) = \frac{x^2 - 1}{x - 1}$.

x	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$						

$$\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1} \approx \underline{\hspace{2cm}}$$

Is $f(1)$ defined? Does that affect the limit? Explain.

2. Complete the table for $g(x) = \frac{\sin x}{x}$ (radian mode).

x	-0.1	-0.01	-0.001	0.001	0.01	0.1
$g(x)$						

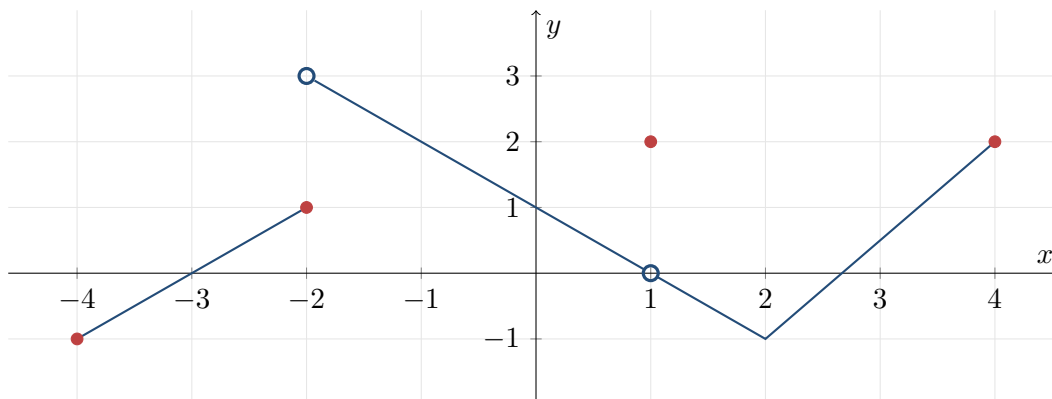
$$\lim_{x \rightarrow 0} \frac{\sin x}{x} \approx \underline{\hspace{2cm}}$$

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2. Read Limits from a Graph

The graph of $y = f(x)$ on $[-4, 4]$ is shown.



Find each value. If a limit does not exist, write DNE and explain why.

$\lim_{x \rightarrow -2^-} f(x)$		$\lim_{x \rightarrow 1} f(x)$	
$\lim_{x \rightarrow -2^+} f(x)$		$f(1)$	
$\lim_{x \rightarrow -2} f(x)$		$\lim_{x \rightarrow 2} f(x)$	
$f(-2)$		$\lim_{x \rightarrow 4^-} f(x)$	
$\lim_{x \rightarrow 0} f(x)$		$\lim_{x \rightarrow -4^+} f(x)$	

1. At which x -values in $(-4, 4)$ does $\lim_{x \rightarrow c} f(x)$ exist but differ from $f(c)$? _____

2. Why does $\lim_{x \rightarrow -2} f(x)$ fail to exist even though $f(-2)$ is defined?

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3. Properties of Limits

If $\lim_{x \rightarrow c} f(x) = L$ and $\lim_{x \rightarrow c} g(x) = M$, then:

Sum/Difference: $\lim_{x \rightarrow c} [f(x) \pm g(x)] = L \pm M$

Product: $\lim_{x \rightarrow c} [f(x) \cdot g(x)] = L \cdot M$

Constant multiple: $\lim_{x \rightarrow c} [k f(x)] = kL$

Quotient: $\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \frac{L}{M}, \quad M \neq 0$

Power: $\lim_{x \rightarrow c} [f(x)]^{r/s} = L^{r/s} \quad (\text{if } L^{r/s} \text{ is real})$

4. Use the Properties

Suppose $\lim_{x \rightarrow 4} f(x) = 3$ and $\lim_{x \rightarrow 4} g(x) = -2$. Find each limit. Name the property you use.

1. $\lim_{x \rightarrow 4} [2f(x) - 5g(x)] =$ _____ Property: _____

2. $\lim_{x \rightarrow 4} [f(x)g(x)]^2 =$ _____ Property: _____

3. $\lim_{x \rightarrow 4} \frac{f(x) + 1}{g(x)} =$ _____ Property: _____

4. $\lim_{x \rightarrow 4} \sqrt{f(x) + 6} =$ _____ Property: _____

5. $\lim_{x \rightarrow 4} [x f(x) + g(x)] =$ _____ Property: _____

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5. Direct Substitution

If f is a polynomial, or a rational function with a nonzero denominator at c , then

$$\lim_{x \rightarrow c} f(x) = f(c)$$

The same is true for $\sin x$, $\cos x$, e^x , $\ln x$, and $\sqrt{x}$ at any c in their domains.

Evaluate each limit.

1. $\lim_{x \rightarrow -2} (x^3 - 4x^2 + 3)$

2. $\lim_{x \rightarrow 3} \frac{x^2 + 1}{2x - 1}$

3. $\lim_{x \rightarrow \pi/3} (2 \sin x + \cos x)$

4. $\lim_{x \rightarrow 1} \sqrt{5x^2 + 4}$

5. $\lim_{x \rightarrow 0} \frac{e^x + \cos x}{2 - x}$

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6. Indeterminate Form $\frac{0}{0}$: Factor and Cancel

If direct substitution gives $\frac{0}{0}$, the limit may still exist. Factor, cancel, and then substitute.

1.

$$\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3}$$

2.

$$\lim_{x \rightarrow 2} \frac{x^3 - 8}{x - 2}$$

3.

$$\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{x^2 - 4}$$

4.

$$\lim_{x \rightarrow 2} \frac{\frac{1}{x} - \frac{1}{2}}{x - 2} \quad (\text{Hint: combine the fractions in the numerator.})$$

5.

$$\lim_{h \rightarrow 0} \frac{(3 + h)^2 - 9}{h}$$

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7. Indeterminate Form $\frac{0}{0}$: Rationalize

Multiply by the conjugate to eliminate the square root, then simplify.

1.

$$\lim_{x \rightarrow 0} \frac{\sqrt{x+4} - 2}{x}$$

2.

$$\lim_{x \rightarrow 9} \frac{\sqrt{x} - 3}{x - 9}$$

3.

$$\lim_{x \rightarrow 4} \frac{2 - \sqrt{x}}{4 - x}$$

4.

$$\lim_{x \rightarrow 1} \frac{x - 1}{\sqrt{x+3} - 2}$$

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8. One-Sided Limits

$$\lim_{x \rightarrow c} f(x) = L \quad \text{if and only if} \quad \lim_{x \rightarrow c^-} f(x) = L \quad \text{and} \quad \lim_{x \rightarrow c^+} f(x) = L$$

1. Let

$$f(x) = \begin{cases} x^2 + 1, & x < 1 \\ 4 - x, & x \geq 1 \end{cases}$$

Find $\lim_{x \rightarrow 1^-} f(x)$, $\lim_{x \rightarrow 1^+} f(x)$, and $\lim_{x \rightarrow 1} f(x)$.2. Find $\lim_{x \rightarrow 0^-} \frac{|x|}{x}$ and $\lim_{x \rightarrow 0^+} \frac{|x|}{x}$. Does $\lim_{x \rightarrow 0} \frac{|x|}{x}$ exist?3. The *greatest integer function* $\lfloor x \rfloor$ gives the largest integer less than or equal to x . Find $\lim_{x \rightarrow 2^-} \lfloor x \rfloor$, $\lim_{x \rightarrow 2^+} \lfloor x \rfloor$, and $\lim_{x \rightarrow 2.5} \lfloor x \rfloor$.4. Find the value of k so that $\lim_{x \rightarrow 2} g(x)$ exists, where

$$g(x) = \begin{cases} x^2 + k, & x < 2 \\ 3x - 1, & x \geq 2 \end{cases}$$

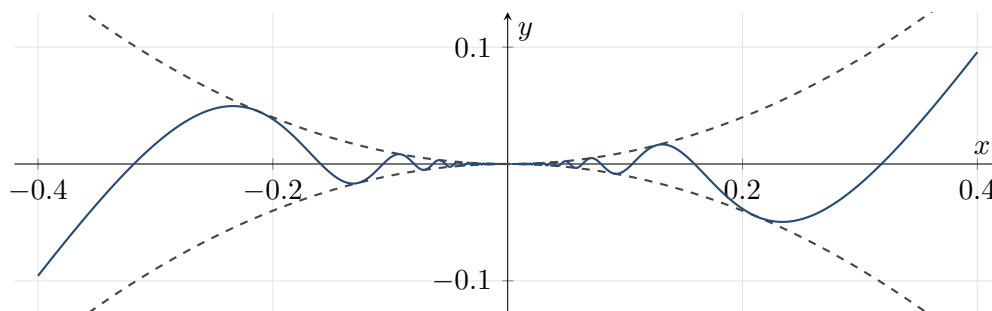
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9. The Sandwich Theorem

If $g(x) \leq f(x) \leq h(x)$ for all $x \neq c$ near c , and $\lim_{x \rightarrow c} g(x) = \lim_{x \rightarrow c} h(x) = L$, then $\lim_{x \rightarrow c} f(x) = L$.

$y = x^2 \sin(1/x)$ trapped between $y = -x^2$ and $y = x^2$



1. Use the Sandwich Theorem to show that $\lim_{x \rightarrow 0} x^2 \sin\left(\frac{1}{x}\right) = 0$.

2. If $1 - x^2 \leq u(x) \leq 1 + x^2$ for all x , find $\lim_{x \rightarrow 0} u(x)$. Explain.

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10. Special Trigonometric Limits

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \qquad \lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0 \qquad (x \text{ in radians})$$

Evaluate each limit. Rewrite each one so that it uses a special limit.

1. $\lim_{x \rightarrow 0} \frac{\sin(3x)}{x}$

2. $\lim_{x \rightarrow 0} \frac{\tan x}{x}$

3. $\lim_{x \rightarrow 0} \frac{\sin(5x)}{\sin(2x)}$

4. $\lim_{x \rightarrow 0} \frac{x}{\sin x}$

5. $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$ (Hint: multiply by $1 + \cos x$.)

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11. When Limits Do Not Exist

A limit fails to exist when the one-sided limits differ, when the function grows without bound, or when it oscillates.

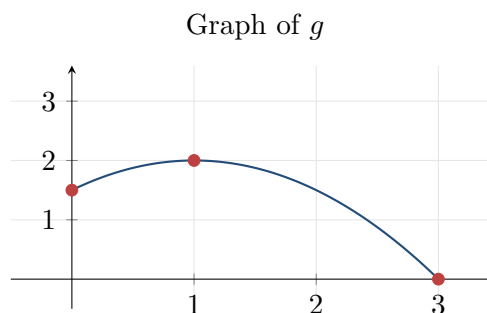
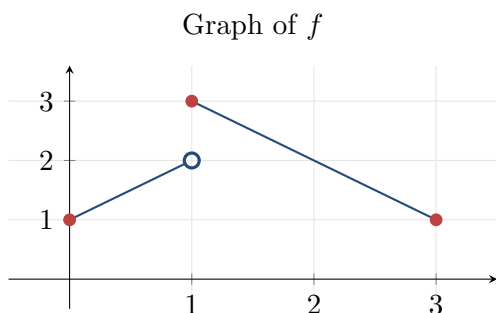
1. Explain why $\lim_{x \rightarrow 0} \frac{1}{x^2}$ does not exist as a real number.

2. Explain why $\lim_{x \rightarrow 0} \sin\left(\frac{1}{x}\right)$ does not exist. (Hint: what happens to $\frac{1}{x}$ as $x \rightarrow 0$?)

3. Explain why $\lim_{x \rightarrow 3} \frac{x-3}{|x-3|}$ does not exist.

12. AP-Style Problem: Limits from Two Graphs

The graphs of f and g are shown.



Find each limit, or explain why it does not exist.

1. $\lim_{x \rightarrow 1^-} [f(x) + g(x)]$ and $\lim_{x \rightarrow 1^+} [f(x) + g(x)]$

2. $\lim_{x \rightarrow 1} [f(x) + g(x)]$ $\lim_{x \rightarrow 2} [f(x) \cdot g(x)]$

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Mixed Review

13. Mixed Practice

Show your work clearly.

1. $\lim_{x \rightarrow -1} \frac{x^2 - x - 2}{x + 1}$

2. $\lim_{x \rightarrow 0} \frac{\sqrt{9+x} - 3}{x}$

3. $\lim_{x \rightarrow 0} \frac{\sin(4x)}{3x}$

4. $\lim_{x \rightarrow 5^-} \frac{|x - 5|}{x - 5}$

14. Explain Your Thinking

Does the value of $f(c)$ affect $\lim_{x \rightarrow c} f(x)$? Give an example to support your answer.

15. Challenge Problem

Find the constants a and b so that $\lim_{x \rightarrow 0} \frac{\sqrt{ax+b} - 2}{x} = 1$.

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16. Graphing Challenge

Sketch the graph of a function f on $[-4, 4]$ that satisfies all of the following:

- $\lim_{x \rightarrow -2} f(x) = 1$, but $f(-2) = 3$
- $\lim_{x \rightarrow 1^-} f(x) = 2$ and $\lim_{x \rightarrow 1^+} f(x) = -1$
- $f(1) = -1$ and $\lim_{x \rightarrow 3} f(x) = f(3) = 0$



Use open and closed circles where needed. Which conditions describe a removable discontinuity, and which describe a jump?

Reflection: What is one thing you understand better about limits after completing this worksheet?
